ANALYSIS AND EMULATION OF EARLY DIGITALLY-CONTROLLED OSCILLATORS BASED ON THE WALSH-HADAMARD TRANSFORM

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ABSTRACT

Early analog synthesizer designs are very popular nowadays, and the discrete-time emulation of voltage-controlled oscillator (VCO) circuits is covered by a large number of virtual analog (VA) textbooks, papers and tutorials. One of the issues of well-known VCOs is their tuning instability and sensitivity to environmental conditions. For this reason, digitally-controlled oscillators were later introduced to provide stable tuning. Up to now, such designs have gained much less attention in the music processing literature. In this paper, we examine one of such designs, which is based on the Walsh-Hadamard transform. The concept was employed in the ARP Pro Soloist and in the Welson Syntex, among others. Some historical background is provided, along with a discussion on the principle, the actual implementation and a band-limited virtual analog derivation.

1. INTRODUCTION

Voltage-controlled oscillators (VCO) are considered among the fundamental building blocks in subtractive synthesis, together with voltage-controlled filters and amplifiers. Before the inception of transistor technology, sound sources in electronic music were very disparate: from heterodyne mixers (e.g., the Theremin), to fixed-frequency electromechanical oscillators (e.g., the Hammond organ), from neon tube oscillators (e.g., the Trautonium) to magnetic tapes. The inception of transistor technology, however, induced early electronic music pioneers to investigate new solutions. Between 1959 and 1960, Harald Bode, a German Engineer that had previously worked for the Cologne studio with Stockhausen, developed a novel concept of modular synthesizer, employing transistor technology [1] and the voltage control paradigm. This system was only meant for sound processing and had no oscillator. Robert Moog later adopted this modular concept to develop what would arguably be the most well-known brand of synthesizers [2]. For his oscillators he considered the 1V per octave paradigm [3], which has later become one of the industry standards. At the beginning of the 1970s many synthesizers were produced, which were based on VCOs.

One of the issues with VCOs is tuning stability. During the 1970s, solutions were proposed for VCOs with better stability, one of which is the shift to Digitally Controlled Oscillators (DCO). Early synthesizer DCO designs were adapted from transistor organs, where usually, a master clock source is divided to obtain the 12 notes of the equal temperament, and from these, a top-octave circuit divides the frequency by multiples of 2 to obtain the lower octaves, all perfectly tuned. These were found on early polyphonic instruments such as string machines and the like.

DCOs became widespread with the growth of the polyphonic synthesizers market, replacing VCOs to reduce pitch drift. With the advent of novel synthesis techniques such as frequency modulation, wavetable, sampling, and physical modelling, the interest in analog VCOs and DCOs was lost. In the 1990s a novel class of digital synthesizers brought back the interest for subtractive synthesis. Research work on *virtual analog* models for oscillators and filter were devised [4, 5], which mostly dealt with alias suppression or faithful recreation of the behavior of analog circuits. From that moment upwards, a great attention has been devoted to such a topic by the research community [6, 7].

Up to now, the literature has dealt mainly with two issues in virtual analog oscillators: aliasing in the generation of geometrical waveforms (e.g., sawtooth or square), as well as analysis and emulation of specific circuits. Since the inception of virtual analog, several techniques have established to generate geometrical waveforms, namely BLIT [4], BLEP [8] (and variations thereof), BLAMP [9], DPW [10] and wavetable synthesis. Other studies addressed the peculiar behavior of analog circuits and their departure from the ideal behavior. This is true for filters, often exhibiting a nonlinear behavior [11, 12], as well as for oscillators departing from the ideal waveform, as it is the case of the Moog sawtooth [13]. Investigating the specificity of existing oscillator designs allows the community to obtain useful information on the timbre of a known instrument, improve its emulation, and verify the applicability of existing aliasing suppression techniques on novel problems.

To the best of authors' knowledge, to date, the virtual analog community has overlooked the study of musical synthesizers' DCOs. DCOs are generally considered less appealing to the musician and the sound designer, because of their supposed precision. For the same reason, they are expected to be of less interest to the researcher as well, as they cause fewer issues in the modelling. Nonetheless, investigating DCO-based synthesizers may bring new insights on the character of these synthesizers, may improve our engineering knowledge, help understand its historical development and revamp some ideas.

This work is concerned with a class of DCO designs based on the Walsh-Hadamard transform. The use of such a transform was appealing for commercial products due to the tuning stability of digital integrated circuits. In the academic literature, the use of this transform for musical purposes was first proposed in a 1973 paper from Bernard Hutchins [14], who described a synthesizer system based on Walsh functions for generating waveforms and envelopes. In his work, Hutchins briefly hinted at its suitability to subtractive synthesis, but focused on additive generation of harmonic and nonharmonic tones. He also acknowledges a colleague,

C. Frederick, for suggesting the very idea of using the Walsh functions for music synthesis. Later works discuss waveform generation, circuit designs, frequency shifting and other purposes of this technique [15, 16].

The remainder of the paper is organized as follows. Section 2 provides a broad definition of DCO and discusses a class of DCOs based on the Walsh-Hadamard transform. Section 3 discusses the peculiar implementation of this method in the Welson Syntex synthesizer and a band-limited virtual analog implementation is proposed in Section 4. Finally, conclusions are drawn in Section 5.

2. DIGITALLY CONTROLLED OSCILLATORS

The definition of a DCO is rather fuzzy. In principle, any oscillator with pitch control acted by digital circuits is a DCO. The term "digital" should not bear confusion with a discrete-time domain oscillator, usually called numerically controlled oscillator (NCO). A NCO is directly implemented in the discrete-time domain, typically to generate a sine wave, and then fed to a Digital to Analog Converter (DAC). The term NCO is generally used in the electronics and telecommunications jargon, but any virtual analog oscillator conforms to that term, since the generation is all numerical.

The term Direct Digital Synthesis is somewhat related to NCO. This technique employs a NCO, often reading an arbitrary waveform from a RAM and generates an analog signal by a DAC. A DCO, instead, is not based on discrete-time algorithms or processing units, but simply works with digital electronics in the continuous-time domain. As an example, whereas a sawtooth VCO accumulates an electric charge into a capacitor and suddenly discharges when a threshold related to an electrical value is reached, a sawtooth DCO discharges at the reaching of a threshold of a digital counter integrated circuit (IC). The sound is, thus, still generated in the continuous-time domain, but the timing is controlled digitally by stable clocks and glue logic. Until digital signal processors, DACs and the production of custom digital VLSI chips became widespread in consumer electronics, the DCO approach was easier and more economical to implement into a synthesizer.

2.1. Walsh-Hadamard DCOs

Synthesizer waveforms based on the Walsh-Hadamard transform can be generated using digital electronics. Although, in principle, other waveforms such as sine waves can be synthesized [14], vintage synthesizers circuits focused on the sawtooth waveform relying on the assumption that it can be decomposed into a sum of square wave signals (with 50% duty cycle) weighted by Walsh-Hadamard transform (WHT) coefficients [17]. This transform reduces a real discrete signal to a weighted sum of orthogonal basis functions. These are the so-called Walsh functions, or Hadamard functions, depending on their ordering. In the following we consider the Walsh ordering.

The $M \times M$ matrix of Walsh functions up to order M is defined as

$$\boldsymbol{W}^{(M)} = \frac{1}{M-1} \cdot (-1)^{\sum_{m=0}^{M} k_m x_{m+1}} \tag{1}$$

where

$$k = \sum_{m=0} k_m 2^m \in \mathbb{N}_0, x = \sum_{m=1} x_m 2^{-m} \tag{2}$$

and both $k_m, x_m \in [0, 1]$.

As an example, the Walsh matrix of order 16 is shown in Figure 1. The WHT of a real-valued row vector $\boldsymbol{x}$ of length M is then

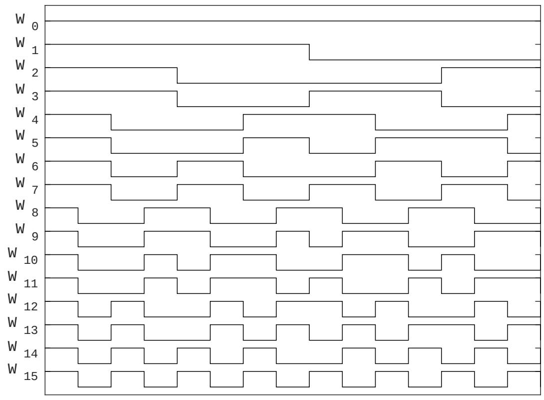

Figure 1: Walsh matrix of order $M = 16$, plotted row-wise. The signals constitute orthogonal bases that are employed in the Walsh transform. The first line corresponds to Walsh function of order 0 (DC), or W_0, while the last has index $M - 1$.

defined as

$$\boldsymbol{X} = \boldsymbol{x} \cdot \boldsymbol{W}^{(M)} \tag{3}$$

The coefficients $\boldsymbol{X}$ correspond to the weights of the orthogonal bases, that allow, thus, resynthesis of the original signal as

$$\hat{\boldsymbol{x}} = \boldsymbol{X} \cdot \boldsymbol{W}^{(M)}. \tag{4}$$

The WHT of a ramp of length M, i.e., a sawtooth of period M, has non-zero coefficients only for odd Walsh functions. For $M = 16$, e.g., only Walsh functions 1, 3, 7 and 15 are non-zero and have coefficients $1, 1/2, 1/4, 1/8$. Digital electronics allows to generate stable square wave signals at a low cost, making this solution viable to generate a sawtooth wave approximation.

3. THE WELSON SYNTEX AND THE ARP PRO SOLOIST

3.1. Historical Background

The Welson Syntex (1976) and the ARP Pro Soloist (1972) were monophonic preset synthesizers of the analog era, similar to the Moog Satellite (1973), the Thomas Synti 1055 and the ARP Soloist (1970). This breed of synthesizers was devised for easy operation on top of other polyphonic instruments such as organs and pianos and generally had limited flexibility. The presets were generally factory hardwired *patches* made of resistor networks that replaced potentiometers to provide fixed values to the synthesizer oscillators, filters, envelope generators, etc. A manual mode was also available where the user could tweak a few parameters regulated by potentiometers on the front panel.

The first successful preset synthesizer was the ARP Pro Soloist (1972), replacing the earlier ARP Soloist that had a limited success, mainly due to tuning stability issues. The ARP Pro Soloist developers devised stable oscillators based on digital electronics. This preset synthesizer had a good success, due to its price, making established brands like Moog eager to add similar products to their product catalogue. The Farfisa Syntorchestra, the Elka Soloist, the Korg 900PS, the Thomas Synti and the Moog Satellite are all similar from a user experience point of view. Many of these were

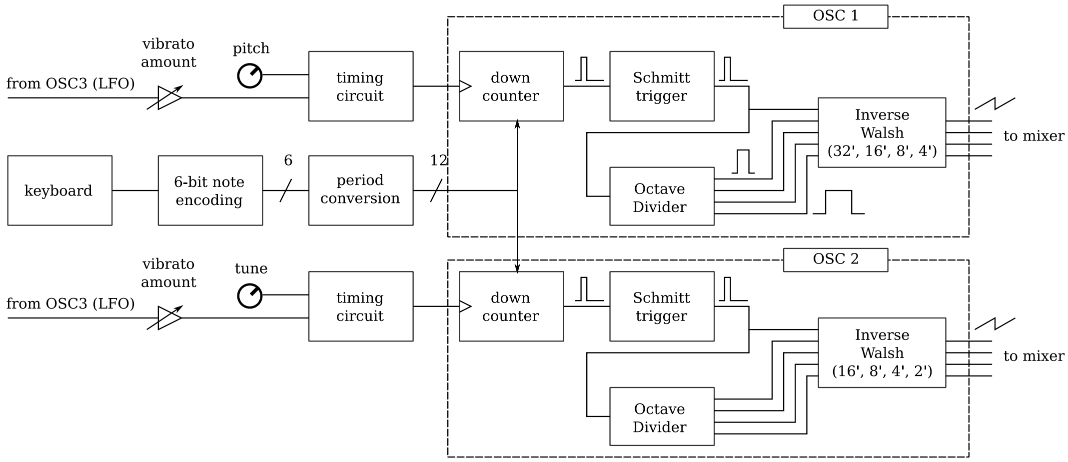

Figure 2: Overview of the Welson Syntex sawtooth generation mechanism.

produced in the Marche region, in Italy. The Farfisa had its main facilities in Camerano, Ancona province, while the Moog Satellite and the Thomas Synti were produced by EME (Elettronica Musicale Europea), a venture of musical instrument manufacturers namely Vox, Thomas and Eko located in Montecassiano, Macerata province. Similarly, the Welson Syntex was produced by Webo Electronics in Passatempo di Osimo, Ancona province. The Italian electronic instruments industry was able to provide know-how, materials and production facilities for these kinds of products.

While the Satellite and the Synti were all-analog, the Pro Soloist featured a digitally controlled oscillator to guarantee tuning stability. This is guaranteed by a resistor ladder DAC that converts ROM-stored values to a control voltage fed to a stable VCO generating a timing pulse train. The circuits, thus, convert a digital value into an analog voltage, and then an analog oscillator feeds a series of digital circuits to obtain several square waves. What is interesting about this oscillator design is that the sawtooth wave is obtained by a weighted sum of square waves, following the WHT DCO concept. This seems to be one of the earliest synthesizers employing the technique, resulting in a very good approximation of a sawtooth wave, as later discussed. Its development may have started a bit earlier than the work in [14], however we have no further information to assess whether the two approaches were developed independently.

The approach was later taken further by the Welson Syntex developers, that greatly reduced the complexity of the circuitry, only employing logic circuits between the keyboard and the oscillator. They also reduced the complexity of the oscillator, obtaining a far from perfect sawtooth wave but with a distinct character that is worth investigating.

3.2. Welson Syntex Synthesizer Architecture

The Syntex was developed in the years 1974–1976 by Mr. Menchinelli and Mr. Elio Bellagamba who were employees at Welson. Its sound engine follows the traditional VCO-VCF-VCA approach. It has two oscillators, differently from all other preset synthesizers mentioned above and a 4-pole transistor ladder filter with automatic keyboard tracking. It features a third oscillator, which actually is a LFO, and a noise generator. A touch of craze was introduced by a “Random Music” button, denoted by an atom icon, that generates random pitches at the rate of the LFO.

The use of digitally controlled oscillators modifies substantially the design of the keyboard and pitch control circuitry with respect to other synthesizers with a VCO. In vintage instruments, the keyboard is usually a resistor ladder that provides a linear voltage change that is later processed, depending on the oscillator. A voltage-controlled oscillator following the 1V/octave paradigm, e.g., rises its pitch exponentially by one octave in response to an increase of 1V to the input. The Syntex, instead, relies on timing and counter circuits without any analog voltage processing stage. An overview is provided in Figure 2. The keyboard is fed to a series of logic IC and gates that generates a 6-bit code. This is further processed by additional glue logic to obtain a 12-bit binary word that is the period duration in clock cycles.

3.3. Oscillator Design

The binary value is loaded into a chain of three 74191 synchronous counters ICs, hardwired to count downwards and arranged to act as a 12-bit counter. At the reaching of zero the binary value is loaded again and the counting starts again. The reset output of the last counter in the chain is also fed to a Schmitt trigger to generate a pulse Q. This pulse has a short active time, $50\mu s$ and a period 8 times shorter than the 8’ output tone. The pulse is fed to a 7493 binary counter, with 4 outputs, that acts as an octave divider. Four octave signals are generated, Q_A, Q_B, Q_C, Q_D, yielding a total of five octaves including Q, although the signals Q_A, Q_B, Q_C, Q_D are 50% duty cycle square waves. The five signals are summed together with different ratios to obtain four footage output. These are all available and are blended together by front panel potentiometers. Oscillator 1 produces 32’, 16’, 8’ and 4’ tones, while Oscillator 2 produces 16’, 8’, 4’, and 2’ [1] In the following analysis we refer to the notation of Oscillator 1, although similar considerations apply to Oscillator 2.

The weighted sum is performed by means of an inverting summing operational amplifier for each footage output as shown in Figure 3. From the resistor values we can obtain the weights for

[1] The timing pulse Q of Oscillator 2 is run at twice the frequency of that of Oscillator 1. The presence of a tuning potentiometer with a wide range for Oscillator 2, however, allows to tune Oscillator 2 in unison to Oscillator 1.

each of the footages outputs:

$$S_{32} = -(Q_{16} + \frac{5}{11}Q_8 + \frac{10}{39}Q_4 + \frac{5}{41}Q_2) \quad (5)$$

$$S_{16} = -(Q_8 + \frac{5}{11}Q_4 + \frac{10}{39}Q_2 + \frac{5}{28}Q) \quad (6)$$

$$S_8 = -(Q_4 + \frac{5}{11}Q_2 + \frac{10}{27}Q) \quad (7)$$

$$S_4 = -(\frac{6}{5}Q_2 + \frac{4}{5}Q) \quad (8)$$

In the Syntex oscillators, four square waves are available, corresponding to Walsh functions of order 1, 3, 7 and 15. Signal Q, in general, is not orthogonal to the others, given the fact that its duty cycle varies with the note pitch. It cannot, therefore, be explained in the light of the WHT transform. The use of Q has been explained by one of its developers, Mr. Elio Bellagamba, as a trade-off between costs and benefits to add a fifth octave wave without resorting to additional components. The musical experts in the company approved this as the produced sound was more aggressive, Mr. Bellagamba recalls.

Regarding the weights of the WHT that are implemented in the Syntex, the chosen discrete resistor values depart from the theoretical values. Signal S_{32} is composed of all square wave signals, thus, it could approximate the sawtooth using a WHT of order 16 if the following resistor values would be employed:

$$S_{32} = -(Q_{16} + \frac{1}{2}Q_8 + \frac{1}{4}Q_4 + \frac{1}{8}Q_2). \quad (9)$$

However, weights do differ in the actual implementation, as seen in Eq. 5. This choice was motivated by the higher cost of precise resistors. Furthermore, the other footages deviate from the ideal WHT formulation as signal Q is not orthogonal to the others. In this case resistor values were agreed with the musical experts. The result of the four outputs are shown in Figure 4, and compared to the measured waveforms.

3.4. Comparison with the ARP Pro Soloist

The ARP Pro Soloist generates the approximated sawtooth waveform by summing 6 square waves generated from a top octave, according to the following:

$$S_{APS} = \sum_{o=1}^{6} \frac{P_o}{2^o}, \quad (10)$$

where P_1 is the square wave with fundamental frequency pitch. These are the weights as required per the WHT to approximate a sawtooth signal. The simplified diagram in Figure 6 shows how the sawtooth wave was obtained.

By employing 6 Walsh functions, the approximation of the sawtooth wave is very good, as shown in Figure 7. The only departure from the ideal sawtooth is the lack of every 64th harmonic, which can be considered negligible, especially for tones over E4 where the 64th harmonic is over the human hearing range. The difference between the ARP Pro Soloist tone and the S_{32} signal from the Welson Syntex is still hardly noticeable. Signals S_{16}, S_8 and S_4, however, depart significantly from the ideal sawtooth tone, making the Welson Syntex oscillator much more interesting to study and model with known virtual analog techniques.

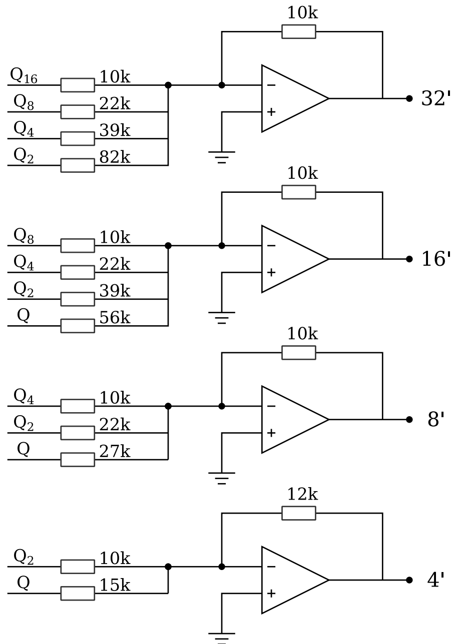

Figure 3: Summing different octave square waves to approximate a sawtooth wave in Welson Syntex Oscillator 1. The lower octave signals Q_2, Q_4, Q_8, Q_{16} are obtained from the top octave signal Q by a binary counter IC (7493) acting as frequency divider, so that Q_2 is half the frequency, Q_4 has one fourth of the frequency and so forth. Please note that the Oscillator 2 shares the same circuit, but the footage outputs are named $16'$ to $2'$ because the timing circuits run at twice the frequency of Oscillator 1, with typical values of the pitch and tuning potentiometers.

4. VIRTUAL ANALOG EMULATION

From the virtual analog side, the generation of the Welson Syntex sawtooth wave is not trivial as the discontinuities can be source of aliasing. In general, there are at least three different strategies to generate the signals seen above:

A: generate the Q_i signals and sum them according to Eqns. 5–8;

B: directly generate a staircase saw;

C: filtering a sawtooth with a comb filter[2].

We shall analyze each of them, their computational cost and their drawbacks. As far as alias suppression is concerned, we shall take the BLEP technique [8] as reference, truncated to 4 samples (2 backward, 2 forward).

Option A is very straightforward in principle. As a drawback, BLEP, or similar alias suppression techniques, need to be applied separately to all five signals. On the other hand, after each of the five signals is generated, all four output footages are obtained with little extra cost. In this case the computational cost per period is $2 + 4 + 8 + 16 + 32$ BLEP, plus the generation of the waves and the weighted sums (10 mul + 9 sums).

Option B allows two solutions: the wave can be directly synthesized by knowing at what point the steps happen and applying

[2]This is not valid for signals S_{16}–S_4 due to the presence of the non-orthogonal signal Q.

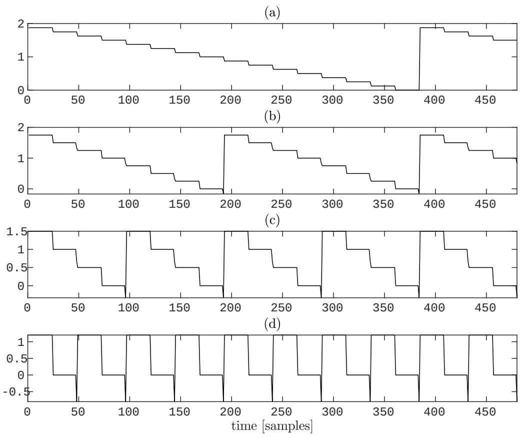

Figure 4: Simulation of the four sawtooth waves from the Welson oscillator: 32' (a) to 4' (d). The duty cycle of the Q signal has been set to 1%.

BLEP at all these discontinuities or by noting that the wave results from the sum of two sawtooth waves, one of larger period and one of shorter period. In other words, a staircase sawtooth S_K with K steps can be generated by the following

$$S_K[n] = S_L[n] - \frac{1}{K} S_H[n] \tag{11}$$

where

$$S_L[n] = 2(n\frac{f_0}{F_s} \mod 1) - 1 \tag{12}$$

$$S_H[n] = 2(n\frac{Kf_0}{F_s} \mod 1) - 1 \tag{13}$$

This requires K+1 BLEP per period, proving very inexpensive if only one of the footage outputs is to be generated. If all outputs need be generated, sawtooth waveforms of period T, $T/2$, $T/4$ and $T/8$ and $T/16$ are generated, costing $1 + 2 + 4 + 8 + 16$ BLEP per period. Finally the Q signal can be obtained by subtracting another sawtooth of period $T/16$, with a phase shift, in order to obtain the $50\mu s$ active time. The overall count is 47 BLEP, less than option A. Figure 8 shows a 2 kHz S_4 signal generated according to option B with and without BLEP.

Option C results from the observation that a sawtooth ramp with K stairs has a null in the spectrum every N harmonics, thus it can be shown that applying a comb filter designed to suppress these harmonics results exactly in a staircase sawtooth. A generalized recursive comb filter with both feedback and feedforward delay lines may be required to filter out the harmonics without affecting the rest of the content. Such a filter is characterized by the following difference equation:

$$y_n = b_0 x_n + b_L x_{n-L} + a_L y_{n-L} \tag{14}$$

where the length of the delay lines is L samples.

The computational cost of this filter is 2 sum and 3 mul per sample and some pointer arithmetics to update the delay lines, which should be added to the cost of 1 BLEP per period to generate the alias-suppressed sawtooth, interpolation of the comb for precise tuning and, possibly the emulation of the Q pulse which should be added to emulate signals S_{16}, S_8, S_4. Overall, this solution is very inexpensive, despite the need to design the comb filter coefficients and to allocate memory for storing the delay lines values. The design of the comb filter may be problematic if, as it is the case with synthesizers, the pitch of the oscillator is modulated. For this reason, options A and B may still be preferable.

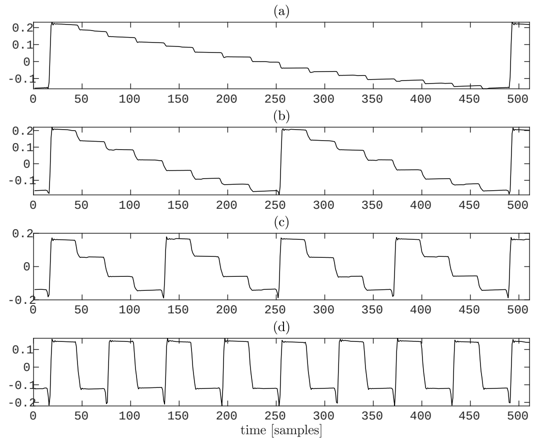

Figure 5: The four sawtooth waveforms sampled from the instrument output with the VCF completely open: 32' (a) to 4' (d). The effect of the sound card DC blocking stage affects the waveform shape by tilting the steps, while some light lowpass filtering due to parasitic components in the analog path slightly smooths the waveforms. This is not seen using an oscilloscope directly at the output of the components.

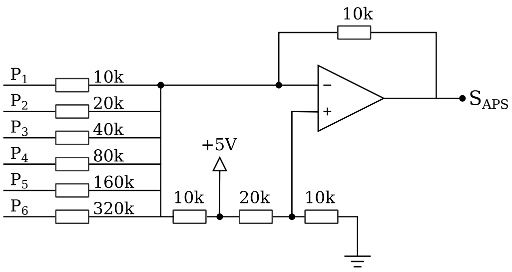

Figure 6: Sawtooth waveform generation in the ARP Pro Soloist. The input square wave signals $P_1, ...P_6$ are generated by a top octave divider. P_1 is the lowest generated octave, corresponding to the note fundamental.

5. CONCLUSIONS

This work described the use of the Walsh-Hadamard transform for sawtooth signals generation and its application in early electronics synthesizers. Two of such oscillator designs, taken from historical synthesizers, have been discussed, and their differences are outlined. These oscillators are classified as DCOs. Therefore, the emulation of their waveforms is not demanding in terms of computational cost or circuit analysis. However, aliasing represents an issue. Several options for virtual analog emulation are described in the paper to obtain very similar results and their computational

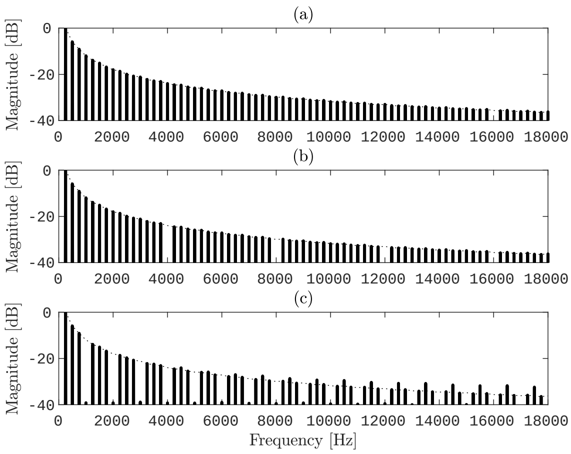

Figure 7: Comparison of ARP Pro Soloist and Welson Syntex sawtooth spectra for tones pitched at 250 Hz. In (a) the ARP Pro Soloist spectrum (solid line) is compared to the ideal sawtooth frequency envelope (dotted line) showing perfect matching (at least up to the 64th harmonic, at 16 kHz). In (b) the S_{32} Welson Syntex spectrum is shown to be identical to (a) except for the lack of each 16th harmonic. In (c) the S_8 signal shows a larger departure from the ideal sawtooth spectral envelope, with every 4th harmonic attenuated. These are not totally canceled due to signal Q not being a 50% duty cycle square wave.

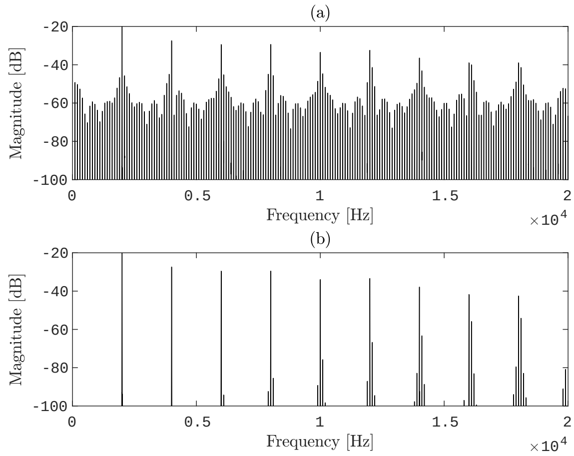

Figure 8: Magnitude spectrum of a 2 kHz S_4 signal generated at 44100 Hz without aliasing suppression (a) and with 4-samples BLEP (b) according to emulation option B.

cost is reported.

Several other synthesizers implemented the WHT for sawtooth generation, making the results of this work useful for the emulation of other historical synthesizers. An example of these is the Korg Poly800, a polyphonic synthesizer from the 1980s. Overall, we argue that DCO designs deserve more interest from the research community both for emulation goals and for preserving good engineering practices. The authors hope that this work could inspire others to study the solutions produced by past engineers, as these represent a rich heritage that may be valuable to progress the state-of-the-art.

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