A ROBUST STOCHASTIC APPROXIMATION METHOD FOR CROSSTALK CANCELLATION

Huaxing Xu Risheng Xia Junfeng Li Yonghong Yan *

Key Laboratory of Speech Acoustics and Content Understanding
Institute of Acoustics
Beijing, China
xuhuaxing, lijunfeng@hccl.ioa.ac.cn

ABSTRACT

Crosstalk cancellation serves as an important role in binaural signals playback through loudspeakers, which reproduce a particular auditory scene to the listener's ears. In practice, due to either the listener's head movement or rotation, etc, the actual transfer function matrix will differ from the design matrix, which results in deterioration in the performance of crosstalk cancellation. Crosstalk cancellation system (CCS) is very non-robust to these perturbations. Generally, in order to improve the robustness of CCS, several pairs of loudspeakers using a multi-band approach processing band-passed content to appropriately spaced loudspeakers are needed. In this paper, by means of assumed stochastic analysis, a stochastic robust approximation method based on random perturbation matrix modeling the variations of the transfer function matrix is introduced and evaluated. Under free-field condition, simulation results demonstrate the effectiveness of the proposed method.

1. INTRODUCTION

Binaural technology is often used to reproduce a virtual auditory scene to the listener as if he/she is personally on the scene. The principle of binaural technology is to reconstruct the acoustic pressures at the listener's eardrums so that the reproduced sound field is identical with what would be produced and can deliver an extremely realistic three dimensional virtual acoustic environment to a listener, which could be of great benefit in virtual reality, augmented reality, computer multimedia, home theater, video games, digital television, and so forth [1, 2]. First, the binaural signals are synthesized by appropriately encoding spatial cues corresponding to the desired target scene, which is suitable for headphone production. In practice, headphone binaural audio production suffers from in-head localization and poor frontal imaging [3], while playback through loudspeakers is largely immune to these problems. In addition, compared with headphone reproduction, cues by the involvement of the listener's own head, torso and pinnae in sound diffraction and reflection during playback can enhance the perceived realism of sound reproduction [4]. When the binaural audio is reproduced through loudspeakers, it suffered from a problem of so-called "crosstalk" component of the signals, i.e. the component of the signal for right ear fed to the left ear and vice versa, which severely destroys the 3D spatial information for the listener.

* This author is also with Xinjiang Laboratory of Minority Speech and Language Information Processing, Chinese Academy of Sciences, Urumqi, Xinjiang

Ideally, the expected signals obtained at the listener's ears are delayed copies of the input binaural signals. To suppress, if not totally eliminate, the unintended crosstalk, in mathematics, it boils down to designing a crosstalk cancellation matrix to approximate the inversion of the transfer function matrix. Since the concept of crosstalk cancellation was introduced in 1960s [5, 6], many studies with the aim of minimizing the crosstalk were extensively investigated [7, 8]. In terms of design of crosstalk cancellation filters, different algorithms have been proposed. In the time domain, the least mean square (LMS) algorithm [7] and its variations [9, 10] are the predominant ones. In contrast to the time-domain method that is time consuming for long filters, the fast frequency-domain deconvolution method offers more advantage in terms of computational speed and is also widely used [8]. Thus far, all of the above-mentioned crosstalk cancellation methods employ the LMS optimization technique. In [11] a method based on a minimax design criterion is proposed, and its solution is obtained by utilizing second-order cone programming (SOCP) techniques. Although it achieves excellent channel separation, especially at low frequencies, its huge computational cost limits its practical applications. In addition, to efficiently implement the crosstalk cancellation system, a number of filter topologies, such as recursive and shuffler form [3, 12, 13], are also presented. Generally, the crosstalk cancellation system is optimized to achieve optimum cancellation at a given transfer function matrix corresponding to a nominal listener's position. However, in practical applications, many factors that disturb the transfer function matrix are unavoidable, such as tiny movements or rotations of the listener's head, noise, etc. All these disturbances or errors have adverse effects on crosstalk cancellation system (CCS), especially when CCS is ill-conditioned. The inverse filter is very sensitive to small errors in the transfer function matrix and may reproduce large distortions in the filter's output. To improve the robustness of CCS, a circular or linear array is suggested using a multi-band approach processing band-passed content to appropriately spaced loudspeakers [14, 15].

However, even with such multiple loudspeakers reproduction, design of the crosstalk cancellation filters with some inherent improved robustness is still necessary. This raises the need for dealing with improving the robustness of crosstalk cancellation system against slight disturbances or errors. When the transfer function between the loudspeakers to the ears is characterized by the room impulse response, they are very are very sensitive to spatial mismatch. Under certain circumstances, the transfer function is a stochastic one. In [16], a spatial robust crosstalk cancellation method is proposed in the case of far-field in reverberant environments. Further, a method that jointly handles the three problems of crosstalk, reverberation reduction, and spatial robustness with respect to varying listening positions was proposed in [17].

Still, due to the fact that crosstalk cancellation of room impulse response in a reverberant environment is extremely non-robust, practical crosstalk cancellation system are commonly designed to cancel only the direct-path transfer functions. In this paper, the aim of this study attempts to model of the disturbance of transfer function itself due to movement of the listener's head from a statistical view. A random variable matrix is introduced to characterize the variations of the transfer function matrix between the loudspeakers and the listener's ears on the basis of statistical modeling. Then the traditional crosstalk cancellation problem turns into stochastic robust approximation problem [18]. In this framework, joint least squares optimization crosstalk cancellation method [19, 20] that take multiple positions into account can be treated as special cases, when the transfer function matrix is subject to discrete distribution. Simulation results demonstrate that this method can improve the robustness of crosstalk cancellation, especially when the nominal transfer function matrix is ill-conditioned.

2. CROSSTALK CANCELLATION FORMULATION

This section presents an overview of well-established material about crosstalk cancellation. Fig. 1 shows a geometry diagram of the implementation of crosstalk cancellation system under investigation, in which p_L and p_R denote the left and right input audio signal, respectively, and h_n^L; n = 1, 2; represent the impulse response (IR) from the nth loudspeaker to the left ear (a similar pair of IRs for the right ear, for concision, are not shown).

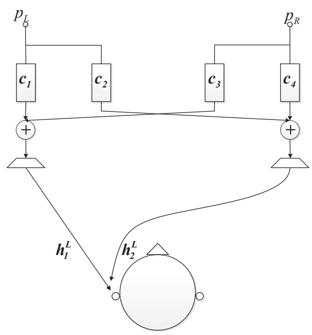


Figure 1: *Geometry diagram for a typical crosstalk cancellation system.*

Considering reproducing only the left audio signal, i.e., p_R = 0, in matrix form, it can be written as

$$\begin{bmatrix} \hat{b}_L \\ \hat{b}_R \end{bmatrix} = \begin{bmatrix} A_1^L & A_2^L \\ A_1^R & A_2^R \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \tag{1}$$

where $\hat{b}_L$ and $\hat{b}_R$ are the transfer functions between p_L and the listener's ears, respectively, and A_1^L is a convolution matrix, which is expressed as

$$A_1^L = \begin{bmatrix} h_1^L(0) & 0 & \cdots & 0 \\ h_1^L(1) & h_1^L(0) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & h_1^L(M-1) \end{bmatrix} \tag{2}$$

and similarly for A_2^L ; A_1^R and A_2^R, and c_1, c_2 are the corresponding crosstalk cancellation filter coefficients vectors. A more simplified form in matrix can be expressed as

$$Ac = b \tag{3}$$

where the transfer function matrix A is composed of A_1^L, A_2^L; A_1^R and A_2^R.

The widely used criterion for crosstalk cancellation is least mean squares (LMS), which minimizes the squared distance between the set of desired input signals and the actual obtained signals at the listener's ears. In our case, b_L is a pure delay, and b_R is a zero vector. For a given head position, CCS filter coefficients can be solved by

$$J_0(c) = \|b - Ac\|_2^2 \tag{4}$$

The optimum filter coefficients are then expressed as

$$c_{opt} = \arg\min_c J_0(c) = A^\dagger b \tag{5}$$

where $A^\dagger = (A^T A)^{-1} A^T$ is the Moore-Penrose generalized inverse of real-valued A. Obviously, such crosstalk cancellation system is only effective when the listener is in the prescribed position and the so-called "sweet spot" is small.

3. PROPOSED STOCHASTIC ROBUST CROSSTALK CANCELLATION METHOD

In practice, the transfer function matrix A is unavoidably influenced by some perturbations and errors due to misalignments, tiny head movement. etc. In this section, we consider the statistical model for the variations in A from statistics point of view.

3.1. Stochastic Robust Approximation

Assuming that A is a random variable matrix taking values in $R^{m\times n}$ with mean $\bar{A}$, A can be expressed as $A = \bar{A} + U$, where U is a random matrix with zero mean. Here, the constant matrix $\bar{A}$ represents the average value of A, and U characterises its statistical variation. Naturally, employing the expected value as the objective function, we can get

$$\arg\min_c E\{\|Ac - b\|\} \tag{6}$$

where E represents the mathematical expectation. This problem is referred to as the stochastic robust approximation problem [21]. When A is a discrete random variable with only a finite number of values, i.e

$$prob(A = A_i) = p_i, i = 1, ..., k \tag{7}$$

where $prob$ means the probability of different $A_i \in R^{m\times n}$, $\mathbf{1}^T p = 1$, $p \succeq 0$, the problem turns into

$$\arg\min_c (p_1\|A_1 c - b\| + ... + p_k\|A_k c - b\|) \tag{8}$$

Therefore, both the joint multi-position optimization [19] and multi-position weighted optimization [20] for crosstalk cancellation can be seen as special cases of the stochastic robust approximation, given by equation (8). Considering the LMS norm, the stochastic LMS method for crosstalk cancellation can be described as

$$\arg\min_c E\{\|Ac - b\|_2^2\} \tag{9}$$

Further, it can be expanded as

$$\begin{aligned} E\{\|Ac-b\|_2^2\} &= E\{(\bar{A}c-b+Ux)^T(\bar{A}c-b+Ux)\} \\ &= (\bar{A}c-b)^T(\bar{A}c-b)+E\{c^TU^TUc\} \\ &= \|\bar{A}c-b\|_2^2+c^TPc \end{aligned} \quad (10)$$

where $P = E\{U^TU\}$ corresponds to mathematical expectation of the autocorrelation matrix of the perturbation matrix U. Therefore the statistical robust approximation problem shows a similar form with the regularized least-squares method [22]

$$\arg\min_{c} \|\bar{A}c-b\|_2^2 + \|P^{1/2}c\|_2^2 \quad (11)$$

with analytical solution

$$c_{opt} = (\bar{A}^T\bar{A}+P)^{-1}\bar{A}^Tb \quad (12)$$

3.2. Modeling Random Perturbation

In the following, the variations of the transfer function are modeled in a way to improve the spatial robustness of crosstalk cancellation from a statistical point of view. Without loss of generality, the perturbation $\xi_i^L(i=1,2)$ on the transfer function from the loudspeakers to listener's left ear is modeled as a statistical variable with zero mean and variance $\sigma_i^L(i=1,2)$ (modeling $\xi_i^R(i=1,2)$ similarly). Then, the perturbed transfer function is expressed as $u_i^L = \xi_i^L h_i^L(i=1,2)$. Further, the perturbation matrix U is denoted as

$$U = \begin{bmatrix} \xi_1^L\bar{A}_1^L & \xi_2^L\bar{A}_2^L \\ \xi_1^R\bar{A}_1^R & \xi_2^R\bar{A}_2^R \end{bmatrix} \quad (13)$$

The expectation matrix P of autocorrelation of the perturbation matrix U is expressed as

$$\begin{aligned} P &= E\{U^TU\} \\ &= E\{\begin{bmatrix} \xi_1^L\bar{A}_1^L & \xi_2^L\bar{A}_2^L \\ \xi_1^R\bar{A}_1^R & \xi_2^R\bar{A}_2^R \end{bmatrix}^T \begin{bmatrix} \xi_1^L\bar{A}_1^L & \xi_2^L\bar{A}_2^L \\ \xi_1^R\bar{A}_1^R & \xi_2^R\bar{A}_2^R \end{bmatrix}\} \\ &= \begin{bmatrix} P_1^L & P_2^L \\ P_1^R & P_2^R \end{bmatrix} \end{aligned} \quad (14)$$

where $P_1^L = E\{(\xi_1^L)^2(\bar{A}_1^L)^T(\bar{A}_1^L)+(\xi_1^R)^2(\bar{A}_1^R)^T(\bar{A}_1^R)\}$, P_2^L, P_1^R, P_2^R denoted similarly.

Due to its uncertainty in practice, it's reasonable to further assume that all the perturbation random variables independent and identically distributed (IID) with zero mean and variance σ, and then the antidiagonal block elements P_2^L, P_1^R of the P matrix are reduced to zeros. Finally, the P matrix is expressed as

$$P = \sigma^2 \begin{bmatrix} (\bar{A}_1^L)^T\bar{A}_1^L+(\bar{A}_1^R)^T\bar{A}_1^R & \mathbf{0} \\ \mathbf{0} & (\bar{A}_2^L)^T\bar{A}_2^L+(\bar{A}_2^R)^T\bar{A}_2^R \end{bmatrix} \quad (15)$$

4. EXPERIMENTAL VERIFICATION AND ANALYSIS

In this section, the performance of the proposed stochastic LMS method is compared with the traditional LMS method by simulations under free-field condition.

4.1. Performance Metrics and Experimental Setup

To analyze the crosstalk cancellation performance, the channel separation (CHS) is adopted as the evaluation measure, which is defined as the ratio between the desired signal and the crosstalk signal. In our case, owing to set the input right signal zeros in prior, the signal received by the listener's left ear is the desired signal and the signal received by the listener's right ear is the crosstalk. There, the channel separation is expressed as

$$CHS(k) = 20\log|\frac{b_L(k)}{b_R(k)}| \quad (16)$$

where k denotes different discrete frequencies. The average channel separation is defined as

$$\overline{CHS} = \frac{1}{n_L-n_H+1}\sum_{k=n_L}^{n_H} CHS(k) \quad (17)$$

where n_L and n_H are the entire frequency ranges of interest.

In order to verify the robustness of the proposed method, the slight movement of the listener's head is selected as the perturbation factor among all the perturbation factors. The average channel separations of different listener's head positions are calculated and compared with traditional LMS method. According to human auditory characteristics, usually, the interaural level difference (ILD) servers as a predominant cue at frequencies below 5 kHz, while in higher frequencies, the listener's head, especially the pinna have a dominant effect in sound localization. Due to free-field condition without consideration of the listener's head effects, the frequency range of computing average channel separation is selected between 200-5000 Hz and the frequency sample is selected as 16 kHz. Fig. 2 illustrates the schematic diagram of the listener's head movement. The loudspeakers are separated by a distance of $d_s = 0.1$ m

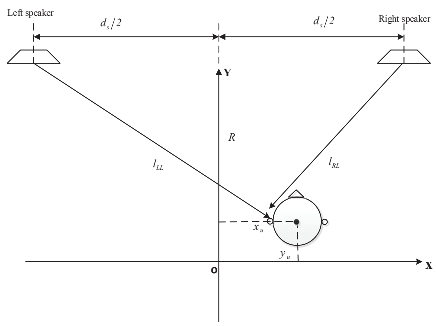


Figure 2: *The schematic of listener's head movement in experiment.*

and an $\theta = 10°$ from the default listening position (with the head placed symmetrically between the loudspeakers) corresponding to the "stereo dipole" configuration [23]. The vertical distance R between the center of the nominal listener's head position to the two speakers is 0.5 m. Because of fundamental difficulties in achieving good crosstalk cancellation at low frequencies, the desired $b_L(n)$ was designed as an unit impulse response filtered by a high-pass filter with a cut-off frequency of 200 Hz. The optimum delay for crosstalk cancellation is calculated according to the rule suggested in [19]. The region x_u for listener's head slight movement is chosen between (1, 2, 3, 4) cm corresponding to the head movement towards the right (the cancellation is more effective as the head moves forwards/backwards than if it moves sideways [24] and further, due to the symmetry only consider the right movement and set y_u = 0). In free field, the transfer function (for example, the left speaker to the left ear) in frequency domain is expressed as

$$H(w) = \frac{1}{4\pi l_{LL}} e^{-jkl_{LL}} \tag{18}$$

where l_{LL} is the distance from the left loudspeaker to the listener's left ear, $k = w/c$ is the wave number and c is the sound speed, set by 340 m/s.

4.2. Analysis of Experimental Result

For proposed stochastic LMS method, the optimal proper σ of the perturbation needs to be determined in advance. A series of tests were conducted by changing the variance σ range (0.01-1) with interval 0.005. For $\theta = 10°$, the average channel separations designed according to the traditional LMS method (solid line) and stochastic LMS method (dashed line) with different σ are shown in Fig. 3. In Fig. 3, the lines from top to bottom describe different head movement positions from x_u = 1 cm to x_u = 4 cm with different line styles represent different methods.

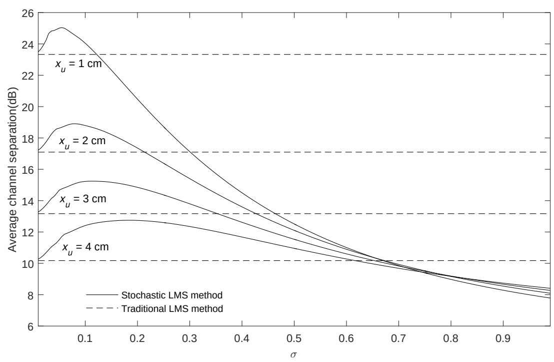


Figure 3: *Comparison of the average CHSs at different head positions: from top to bottom x_u = 1 cm, 2 cm, 3 cm, 4 cm for $\theta = 10°$ with different σ (solid line for the stochastic LMS method and dashed line for the traditional LMS method).*

Strictly speaking, for the traditional LMS method, the average channel separation of different head movement is a specific value and does not vary as the σ varies, which is depicted as a straight line. It's clearly shown from the Fig. 3 that in the vicinity of σ = 0.1, the average channel separation of the proposed stochastic LMS method is higher than the corresponding traditional LMS method, demonstrating that the proposed method is robust against the listener's slight movement. For clearly showing the improvement, the listener's ear responses are drawn in Fig. 4 with different head movements. Ideally, the left ear response should be unity (above 200 Hz) and the right ear response should be zero. As shown from the Fig. 3 and Fig. 4, compared with the traditional LMS method, introducing the perturbation brings improved channel separation in the vicinity of 1000 Hz.

Under the free-field condition, the analysis of the transfer function matrix revealed that, for a given loudspeaker angle, its robust frequency range is determined by the "ring frequency"(RF), which is inversely proportional to the angle [23]. It indicates that the crosstalk cancellation is inherently non-robust in the frequency range above the RF. The RF of the "stereo dipole" is about 11 kHz, which is beyond the scope of the frequency (8 kHz) considered in our experiment. For further comparison, the loudspeaker angle is increased to 20°, where the RF is about 5.6 kHz and repeat the experiment with other parameters keeping the same. Similar to Fig. 3, for $\theta = 20°$, the the average channel separation designed according to the traditional LMS method and stochastic LMS method with different variance σ and head positions are shown in Fig. 5.

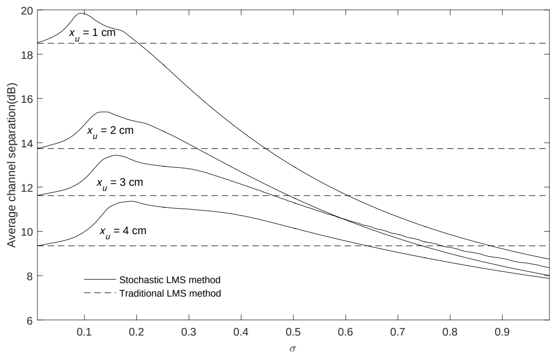
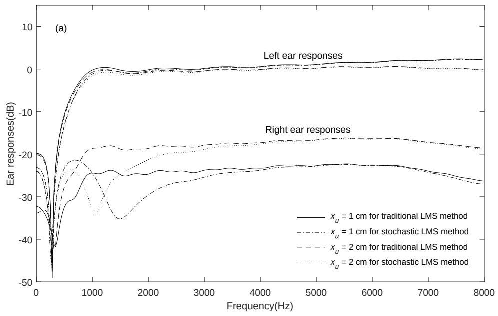


Figure 5: *Comparison of the average CHSs at different head positions: from top to bottom x_u = 1 cm, 2 cm, 3 cm, 4 cm for $\theta = 20°$ with different σ (solid line for the stochastic LMS method and dashed line for the traditional LMS method).*

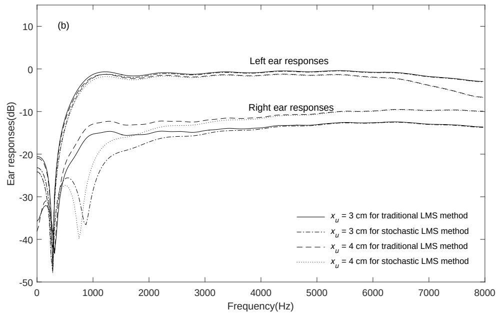
As can be seen from Fig. 5, in the vicinity of σ = 0.15, the average channel separation of the proposed stochastic LMS method is still higher than the traditional LMS method, which shows good agreement with the first experiment. When the variance σ = 0.15, the listener's ear responses are drawn in Fig. 6 with different head movement. From discussions described above, there exists non-robust frequency point corresponding the "ring frequency" around 5000 Hz. The perturbation introduced by the listener's slight head movement results in the rapid decrease of its performance and the spectral distortion. The proposed stochastic LMS method not only improves channel separation in the vicinity of 1000 Hz, but also greatly reduces the spectral distortion around the "ring frequency". This again confirms the expectation that proposed stochastic approximation crosstalk cancellation method provides an enhanced robustness.

5. CONCLUSION

A novel stochastic LMS crosstalk cancellation method based on statistical modeling is proposed for the designing of crosstalk cancellation system. A random perturbation matrix modeling the variations of the transfer functions due to perturbations is introduced and lied in parallel to the actual nominal transfer matrix during driving the crosstalk cancellation filters. Under the free-field condition, simulation results proved that the proposed method is robust against listener's slight head movement.

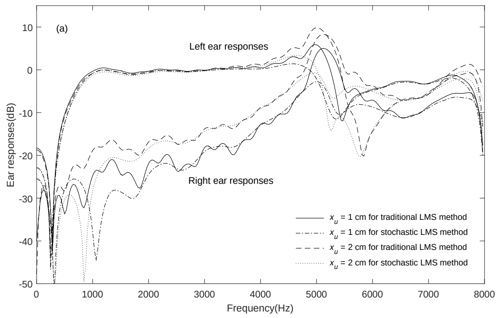


(a)

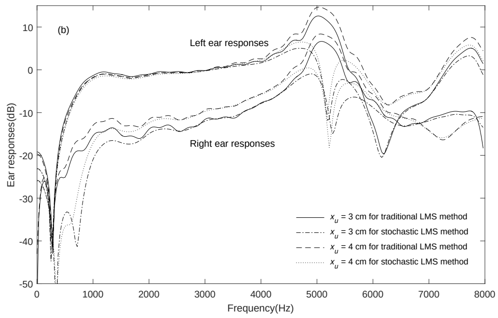


(b)

Figure 4: *Ear responses at head positions with different line styles representing different methods for $\theta = 10°$: (a) x_u = 1 cm, 2 cm; (b) x_u = 3 cm, 4 cm.*



(a)



(b)

Figure 6: *Ear responses at head positions with different line styles representing different methods for $\theta = 20°$: (a) x_u = 1 cm, 2 cm; (b) x_u = 3 cm, 4 cm.*

6. ACKNOWLEDGMENTS

This work is partially supported by the National Natural Science Foundation of China (Nos. 11461141004, 61271426, 11504406, 11590770, 11590771, 11590772, 11590773, 11590774), the Strategic Priority Research Program of the Chinese Academy of Sciences (Nos. XDA06030100, XDA06030500, XDA06040603), National 863 Program (No. 2015AA016306), National 973 Program (No. 2013CB329302) and the Key Science and Technology Project of the Xinjiang Uygur Autonomous Region (No. 201230118- 3).

7. REFERENCES

[1] Yiteng Huang, Jingdong Chen, and Jacob Benesty, "Immersive audio schemes," *Signal Processing Magazine, IEEE*, vol. 28, no. 1, pp. 20–32, 2011.

[2] Bosun Xie, *Head-related transfer function and virtual auditory display*, J. Ross Publishing, 2013.

[3] William G Gardner, *3-D audio using loudspeakers*, vol. 444, Springer Science & Business Media, 1998.

[4] Edgar Y Choueiri, "Optimal crosstalk cancellation for binaural audio with two loudspeakers," *Princeton University*, p. 28, 2008.

[5] Benjamin B Bauer, "Stereophonic earphones and binaural loudspeakers," *Journal of the Audio Engineering Society*, vol. 9, no. 2, pp. 148–151, 1961.

[6] Bishnu S Atal and Manfred R Schroeder, "Apparent sound source translator," Feb. 22 1966, US Patent 3,236,949.

[7] Philip A Nelson, Hareo Hamada, and Stephen J Elliott, "Adaptive inverse filters for stereophonic sound reproduction," *Signal Processing, IEEE Transactions on*, vol. 40, no. 7, pp. 1621–1632, 1992.

[8] Ole Kirkeby, Philip A Nelson, Hareo Hamada, and Felipe Orduna-Bustamante, "Fast deconvolution of multichannel systems using regularization," *Speech and Audio Processing, IEEE Transactions on*, vol. 6, no. 2, pp. 189–194, 1998.

[9] Jong-Soong Lim and Chris Kyriakakis, "Multirate adaptive filtering for immersive audio," in *Acoustics, Speech, and Signal Processing, 2001. Proceedings.(ICASSP'01). 2001 IEEE International Conference on*. IEEE, 2001, vol. 5, pp. 3357–3360.

[10] Athanasios Mouchtaris, Panagiotis Reveliotis, and Chris Kyriakakis, "Inverse filter design for immersive audio rendering over loudspeakers," *Multimedia, IEEE Transactions on*, vol. 2, no. 2, pp. 77–87, 2000.

[11] Harsha IK Rao, V John Mathews, and Young-Cheol Park, "A minimax approach for the joint design of acoustic crosstalk

cancellation filters," *Audio, Speech, and Language Processing, IEEE Transactions on*, vol. 15, no. 8, pp. 2287–2298, 2007.

[12] Makoto Iwahara and Toshinori Mori, "Stereophonic sound reproduction system," Oct. 3 1978, US Patent 4,118,599.

[13] Duane H Cooper and Jerald L Bauck, "Prospects for transaural recording," *Journal of the Audio Engineering Society*, vol. 37, no. 1/2, pp. 3–19, 1989.

[14] Takashi Takeuchi and Philip A Nelson, "Optimal source distribution for binaural synthesis over loudspeakers," *The Journal of the Acoustical Society of America*, vol. 112, no. 6, pp. 2786–2797, 2002.

[15] Jianwen Zheng, Tianyi Zhu, Jing Lu, and Xiaojun Qiu, "A linear robust binaural sound reproduction system with optimal source distribution strategy," *Journal of the Audio Engineering Society*, vol. 63, no. 9, pp. 725–735, 2015.

[16] Markus Kallinger and Alfred Mertins, "A spatially robust least squares crosstalk canceller," in *Acoustics, Speech and Signal Processing, 2007. ICASSP 2007. IEEE International Conference on*. IEEE, 2007, vol. 1, pp. I–177.

[17] Jan Ole Jungmann, Radoslaw Mazur, Markus Kallinger, Tiemin Mei, and Alfred Mertins, "Combined acoustic mimo channel crosstalk cancellation and room impulse response reshaping," *Audio, Speech, and Language Processing, IEEE Transactions on*, vol. 20, no. 6, pp. 1829–1842, 2012.

[18] Stephen Boyd and Lieven Vandenberghe, *Convex optimization*, Cambridge university press, 2004.

[19] Darren B Ward, "Joint least squares optimization for robust acoustic crosstalk cancellation," *Speech and Audio Processing, IEEE Transactions on*, vol. 8, no. 2, pp. 211–215, 2000.

[20] Wang Jie, Ye Qing-hua, Zheng Cheng-shi, and Li Xiao-Dong, "A robust algorithm for binaural audio reproduction using loudspeakers," in *Measuring Technology and Mechatronics Automation (ICMTMA), 2010 International Conference on*. IEEE, 2010, vol. 1, pp. 318–321.

[21] Laurent El Ghaoui and Hervé Lebret, "Robust solutions to least-squares problems with uncertain data," *SIAM Journal on Matrix Analysis and Applications*, vol. 18, no. 4, pp. 1035–1064, 1997.

[22] Ole Kirkeby and Philip A Nelson, "Digital filter design for virtual source imaging systems," in *Audio Engineering Society Convention 104*. Audio Engineering Society, 1998.

[23] Ole Kirkeby, Philip A Nelson, and Hareo Hamada, "The "stereo dipole" : A virtual source imaging system using two closely spaced loudspeakers," *Journal of the Audio Engineering Society*, vol. 46, no. 5, pp. 387–395, 1998.

[24] Darren B Ward and Gary W Elk, "Optimum loudspeaker spacing for robust crosstalk cancellation," in *Acoustics, Speech and Signal Processing, 1998. Proceedings of the 1998 IEEE International Conference on*. IEEE, 1998, vol. 6, pp. 3541–3544.