HARPSICHORD SOUND SYNTHESIS USING A PHYSICAL PLECTRUM MODEL INTERFACED WITH THE DIGITAL WAVEGUIDE

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ABSTRACT

In this paper, we present a revised model of the plectrum-string interaction and its interface with the digital waveguide for simulation of the harpsichord sound. We will first revisit the plectrum body model that we have proposed previously in [1] and then extend the model to incorporate the geometry of the plectrum tip. This permits us to model the dynamics of the string slipping off the plectrum more comprehensively, which provides more physically accurate excitation signals. Simulation results are presented and discussed.

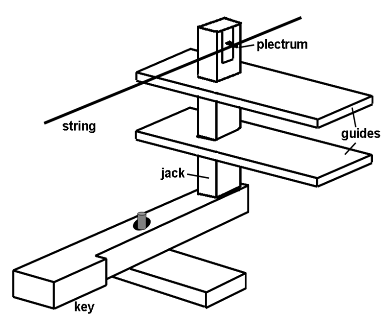

Figure 1: *Harpsichord key and jack.*

1. INTRODUCTION

The harpsichord is a plucked string keyboard instrument which was first invented probably around the late 14th century [2]. A predecessor of the piano, its popularity reached its peak in the 17th century, becoming one of the most important keyboard instruments of the Baroque era. The harpsichord became "obsolete" rather quickly after the maturation of the piano, but the 20th century early music movement has since renewed significant interest towards the instrument. Figure 1 shows the mechanism in which the harpsichord strings are sounded. When the key is played, the harpsichord jack is guided to move vertically upwards and a flexible plectrum mounted at the end of the jack plucks the string.

General harpsichord physics have been discussed in [3, 4, 5] discussing the various components of the harpsichord. More specific studies such as the soundboard vibration modes or attack transients can be found in [6, 7, 8, 9, 10, 11]. The dynamics of the harpsichord, generally thought to be nonexistent, has been studied in greater detail in [12] and has shown actually that a limited amount of dynamics and timbral changes exist.

The interaction between the harpsichord plectrum and string is an aspect much less studied. A theoretical model was first proposed by Griffel [13], prompting further studies and a modified plectrum model proposed by Giordano and Winans II [14]. In contrast, the finger-string interaction has been studied and modeled in more detail in both the guitar [15, 16, 17, 18, 19] and concert harp [20, 21]. For the guitar, differences in radiated sound due to changes in guitar plectrum parameters have been reported in [22, 23], and a guitar plectrum-string interaction model can be found in [24].

A more thorough harpsichord plectrum has been recently proposed by the authors [1, 25], which excites both transverse motions of the strings and allows for interfacing with digital waveguides [26, 27]. In this paper, we extend our model to incorporate the plectrum tip geometry, describing the final stages of the string slipping off the plectrum more completely, important in synthesizing a more accurate string excitation signal. The physical-based excitations provide controllability and expressivity that can complement existing models using a sampled excitation database [28].

In Section 2 we present our improved plectrum model, which includes a review of our previous plectrum body model and the new plectrum tip model. In Section 3 we discuss the plectrum-string interaction, where the interaction at the plectrum body and tip are treated differently. In Section 4, we detail the interfacing between our plectrum model with the digital waveguide. Simulation results are discussed in Section 5, and in Section 6 we draw our conclusions.

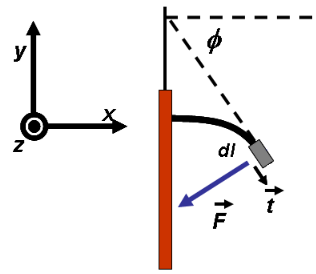

Figure 2: *Force exerted on plectrum.*

2. PLECTRUM MODEL

In this section, we will first review the plectrum body model. We will then extend our plectrum model so that it incorporates the tip geometry of the plectrum, important for an accurate description of the string during slip-off from the plectrum.

2.1. Model Assumptions

For our model, we shall assume that

- the strain is small within the plectrum (still allows for large end deflection)
- the plectrum is an isotropic elastic material
- the plectrum has a uniform rectangular cross section
- the plectrum only bends in a plane so that there is no twisting motion
- there is no friction between the string and plectrum
- the force exerted on the plectrum is always perpendicular to the surface in contact
- the force is concentrated only at one point
- the plectrum mass is ignored, and thus the plectrum and string are assumed to be quasi-static, neglecting any oscillations from the plectrum's inertia.

2.2. Plectrum Body

The harpsichord plectrum is modeled as a thin rectangular rod clamped at one end and free on the other end. When a clamped rod is subject to an external force $\vec{F}$, it results in a bending moment $\vec{M}$ due to the internal stresses. The general equilibrium equation for a bent rod is given by

$$\frac{d\vec{M}}{dl} = \vec{F} \times \vec{t} \tag{1}$$

where dl is an infinitesimal element of the rod, and $\vec{t}$ is a unit vector tangential to the rod. Under the assumptions in the previous section, the bending moment can be written as

$$M = EI \, \frac{d\vec{r}}{dl} \times \frac{d^2\vec{r}}{dl^2} \tag{2}$$

where E is the Young's Modulus, I is the second moment of inertia (or area moment of inertia), and $\vec{r}$ is the radius vector from a fixed point to the point considered on the rod. Defining a coordinate axis such that the *x*-*y* plane denotes the plane of the bent rod and ϕ as the angle between the horizontal and $\vec{t}$, shown in Figure 2, the equation is simplified to

$$EI \, \frac{d^2\phi}{dl^2} + F = 0 \tag{3}$$

Imposing the correct boundary conditions at the free end, we can solve for the deflection angle along the length of the plectrum,

$$\phi(l) = \frac{F}{EI}(Ll - \frac{1}{2}l^2) \tag{4}$$

$$\phi_0 \equiv \phi(l = L) = \frac{1}{2}\frac{FL^2}{EI} \tag{5}$$

where L is length of the plectrum. The parametric shape of the plectrum $x(l)$ and $y(l)$ can be found as

$$\begin{cases} x(l) & = \int \cos\phi \, dl \\ y(l) & = -\int \sin\phi \, dl \end{cases} \tag{6}$$

For small-angle approximations ($\phi \ll 1$), this reduces to the commonly seen cantilever beam loading equations. A bent harpsichord plectrum, however, undergoes significant deflection, and these cantilever beam equations do not agree well with the general solution (6). A revised approximation that the authors have proposed is given by

$$\begin{cases} x(l) & = l - \frac{1}{2}\left(\frac{F}{EI}\right)^2\left(\frac{L^2l^3}{3} - \frac{Ll^4}{4} + \frac{l^5}{20}\right) \\ y(l) & = -\left(\frac{F}{EI}\right)\left(\frac{Ll^2}{2} - \frac{l^3}{6}\right) \\ & \quad +\frac{1}{6}\left(\frac{F}{EI}\right)^3\left(\frac{L^3l^4}{4} - \frac{3L^2l^5}{10} + \frac{Ll^6}{8} - \frac{l^7}{56}\right) \end{cases} \tag{7}$$

which gives good agreement even up to end deflection angles ϕ of 45°.

2.3. Plectrum Tip

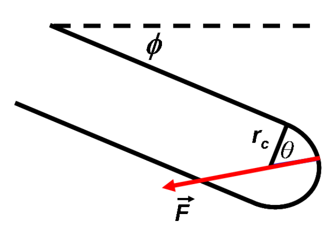

Figure 3: *Force exerted on plectrum tip.*

In order to account for the geometry of the end of the plectrum, we will model the plectrum tip as a circular tip with diameter equal to that of the thickness of the plectrum. We will go through derivations similar to that of the previous section 2.2. Also keeping in mind that an exerted force on the tip of the plectrum must still be perpendicular to the surface, as in Fig 3, equation (3) becomes

$$EI\,\frac{d^2\phi}{dl^2} + F(\cos\theta) = 0 \tag{8}$$

where θ denotes the angle and position on the tip the force is applied. The deflection angles become

$$\phi(l) = \frac{F(\cos\theta)}{EI}(Ll - \frac{1}{2}l^2) \tag{9}$$

$$\phi_0 \equiv \phi(l = L) = \frac{1}{2}\frac{F(\cos\theta)L^2}{EI} \tag{10}$$

Similarly, for the revised approximation of the plectrum shape, all the F terms are replaced with $F(\cos\theta)$. Note that when $\theta = 90^\circ$, there is no bending moment on the plectrum, and the plectrum becomes unbent with zero deflection $\phi(l) = 0$. In the case of the harpsichord plectrum and string, this represents the moment when the string slides past and leaves the plectrum. It is also clear from the figure that θ will not be larger than 90°, as this would imply that the plectrum is bent upwards instead.

3. PLECTRUM-STRING INTERACTION

In this section, we will discuss the interaction between the harpsichord plectrum and string while they are in contact when the string is plucked. Assuming small string displacements, a segment of the string with mass Δm and length Δz that is in contact with the plectrum follows the equations of motion,

$$\begin{cases} (\Delta m)\dfrac{\partial^2 x_s(t)}{\partial t^2} = K\dfrac{\partial^2 x_s(t)}{\partial z^2}(\Delta z) + F_{p\_x} \\[2ex] (\Delta m)\dfrac{\partial^2 y_s(t)}{\partial t^2} = K\dfrac{\partial^2 y_s(t)}{\partial z^2}(\Delta z) + F_{p\_y} \end{cases} \tag{11}$$

where $x_s(t)$ and $y_s(t)$ denote the transverse string segment displacements, K is the tension of the string, F_{p_x} and F_{p_y} are the x and y components of the plectrum force F exerted on the string segment, and z is the coordinate along the string, perpendicular to both $x_s(t)$ and $y_s(t)$. The time when the string is sliding along the main plectrum body and when it is slipping off the tip must be treated differently.

3.1. Sliding Along Plectrum Body

As shown in Figure 4, the clamped end of the plectrum moves with the harpsichord jack, constrained to move only in the vertical direction. Its position is denoted by $(x_j(t),\ y_j(t))$. During the phase where the string is sliding along the plectrum body, the string is at a distance $L' < L$ from the clamped end. Using our revised approximation of equation (7) evaluated at the location of the string $l = L'$, we have

$$\begin{cases} x_s(t) - x_j(t) &= L' - \left(\dfrac{F}{EI}\right)^2 \dfrac{L'^5}{15} \\[2ex] y_s(t) - y_j(t) &= -\left(\dfrac{F}{EI}\right)\dfrac{L'^3}{3} + \left(\dfrac{F}{EI}\right)^3 \dfrac{L'^7}{105} \end{cases} \tag{12}$$

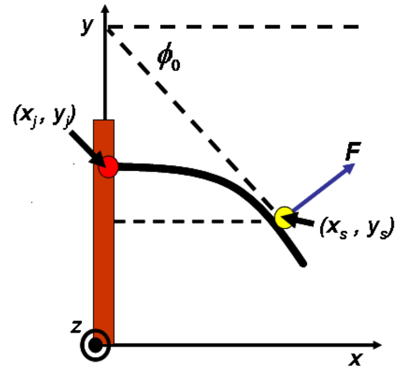

Figure 4: *Plectrum and string interaction along main plectrum body.*

The deflection angle at $l = L'$ is given by equation (5), and therefore the components of the plectrum force are given by

$$\begin{cases} F_{p\_x} &= F\sin\left(\dfrac{FL'^2}{2EI}\right) \\[2ex] F_{p\_y} &= F\cos\left(\dfrac{FL'^2}{2EI}\right) \end{cases} \tag{13}$$

If we know the motion of the harpsichord jack, the transverse string segment displacements can be calculated using equations (11) to (13).

3.2. Slip-off

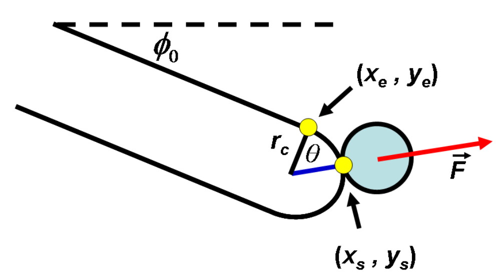

Figure 5: *Plectrum and string interaction at plectrum tip.*

As shown in Figure 5, as the string slides past the end of the plectrum body, labeled in the figure as point $(x_e,\ y_e)$, it proceeds to slip off the tip. While the string is on the plectrum tip, it is a distance $L'' = L + \Delta$ from the clamped end of the plectrum, where Δ is the additional length correction from the tip. However, Δ is on the order of the thickness of the plectrum, which is much smaller than the length of the plectrum. To reduce the complexity of the problem, we will first make the approximation that the force F is applied at the point $(x_e,\ y_e)$. Using the plectrum tip model of section 2.3, the deflection of the plectrum at $(x_e,\ y_e)$ is given by

$$\begin{cases} x_e(t) - x_j(t) &= L - \left(\dfrac{F(\cos\theta)}{EI}\right)^2 \dfrac{L^5}{15} \\ y_e(t) - y_j(t) &= -\left(\dfrac{F(\cos\theta)}{EI}\right)\dfrac{L^3}{3} + \left(\dfrac{F(\cos\theta)}{EI}\right)^3 \dfrac{L^7}{105} \end{cases} \tag{14}$$

From the geometry in Figure 5, we also find that

$$\begin{cases} x_s(t) - x_e(t) &= r_c\left[\sin(\theta+\phi_0) - \sin(\phi_0)\right] \\ y_s(t) - y_e(t) &= r_c\left[\cos(\theta+\phi_0) - \cos(\phi_0)\right] \end{cases} \tag{15}$$

where r_c is the radius of curvature of the tip, equal to half the plectrum thickness. Combining these two expressions with equation (10), we have the plectrum deflection at the location of the string which accounts for the additional length correction of the plectrum tip:

$$\begin{cases} x_s(t) - x_j(t) &= L\left(1 - \dfrac{4\phi_0^2}{15}\right) + r_c\left[\sin(\theta+\phi_0) - \sin(\phi_0)\right] \\ y_s(t) - y_j(t) &= -\dfrac{2L\phi_0}{3}\left(1 - \dfrac{4\phi_0^2}{35}\right) \\ & \quad + r_c\left[\cos(\theta+\phi_0) - \cos(\phi_0)\right] \end{cases} \tag{16}$$

Note that if $\theta = 0$, this expression reduces to equation (14), which represents the string just at the edge of the plectrum body. Similarly, the components of the plectrum force are now

$$\begin{cases} F_{p\_x} &= F\sin(\theta+\phi_0) \\ F_{p\_y} &= F\cos(\theta+\phi_0) \end{cases} \tag{17}$$

and therefore the the transverse string segment displacements can be calculated once again.

4. DIGITAL WAVEGUIDE INTERFACE WITH PLECTRUM MODEL

For segments of the string not in contact with the plectrum, the equations of motion (11) are reduced to the wave equation

$$\begin{cases} \dfrac{\partial^2 x_s(t)}{\partial t^2} = c^2 \dfrac{\partial^2 x_s(t)}{\partial z^2} \\ \dfrac{\partial^2 y_s(t)}{\partial t^2} = c^2 \dfrac{\partial^2 y_s(t)}{\partial z^2} \end{cases} \tag{18}$$

where $c = \sqrt{K/\mu}$ is the string wave propagation speed, K is the string tension defined earlier, and $\mu = (\Delta m)/(\Delta z)$ is the linear mass density. D'Alembert's traveling-wave solution to the wave equation is well-known and can be expressed as

$$\begin{cases} x_s(z,t) &= x^-(z+ct) + x^+(z-ct) \\ y_s(z,t) &= y^-(z+ct) + y^+(z-ct) \end{cases} \tag{19}$$

where x^- and y^- represent the traveling waves in the $-z$ direction and x^+ and y^+ in the $+z$ direction. In the discrete-time domain, traveling waves are simulated efficiently by means of digital

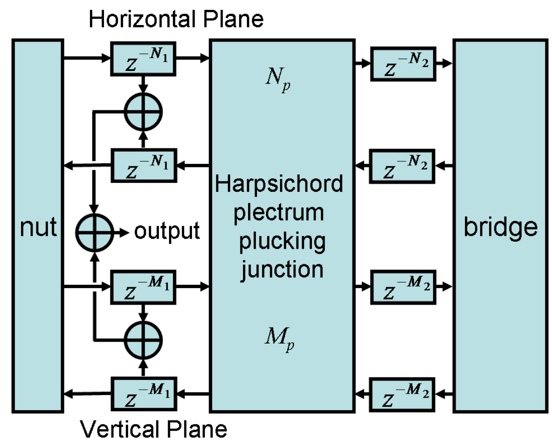

Figure 6: *Harpsichord string synthesis model.*

waveguides. The harpsichord synthesis model is represented by the block diagram in Figure 6. Pairs of digital waveguide delay-lines are implemented on both sides of the plucking junction. In addition, our plectrum model excites both the horizontal and vertical transverse modes of string vibrations, and the two modes can be coupled both at the nut and the bridge.

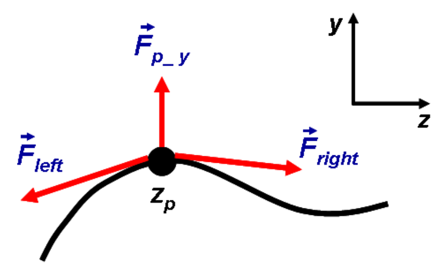

Figure 7: *Sum of forces at the plucking point.*

The length of the segment of string in contact with the plectrum Δz is much smaller than the length of the string L_s and can be effectively reduced to a single point. As shown in Figure 7 for the transverse vertical y component, the equilibrium of the sum of the forces on the plucking point gives

$$\vec{F}_{p\_y} + \vec{F}_{left} + \vec{F}_{right} = 0 \tag{20}$$

For small displacements, the y component of the left and right string forces can be approximated as

$$\begin{cases} F_{left\_y} &\approx -K \left.\dfrac{\partial y_s}{\partial z}\right|_{z_p^-} \\ F_{right\_y} &\approx K \left.\dfrac{\partial y_s}{\partial z}\right|_{z_p^+} \end{cases} \tag{21}$$

As with the traveling wave solution (19) of string displacements, the left and right string forces can also be decomposed as left and right traveling "force waves." Defining the force wave as

$$f^{\mp}(z \pm ct) \equiv -K \frac{\partial y^{\mp}(z \pm ct)}{\partial z} \tag{22}$$

the traveling wave decomposition of the forces gives

$$\begin{cases} F_{left\_y} &= f^{-}_{left}(z+ct) + f^{+}_{left}(z-ct) \\ F_{right\_y} &= -f^{-}_{right}(z+ct) - f^{+}_{right}(z-ct) \end{cases} \tag{23}$$

Further relating the spatial and time partial derivatives of the force waves, we have expressions for the "Ohm's Law" for traveling waves

$$\begin{cases} f^{-} &= -K\dfrac{\partial y^{-}}{\partial z} = -\dfrac{K}{c}\dfrac{\partial y^{-}}{\partial t} = -Rv^{-} \\ \\ f^{+} &= -K\dfrac{\partial y^{+}}{\partial z} = \dfrac{K}{c}\dfrac{\partial y^{+}}{\partial t} = Rv^{+} \end{cases} \tag{24}$$

where $R = K/c = \sqrt{K\mu}$ is the wave impedance of the string, and v^{-} and v^{+} are the traveling velocity wave components of the transverse string velocity. Rewriting equation (20) in terms of the traveling force waves and also using the Ohm's Law for traveling waves,

$$(f^{-}_{left} + f^{+}_{left}) - (f^{-}_{right} + f^{+}_{right}) + F_{p\_y} = 0 \tag{25}$$

$$R(-v^{-}_{left} + v^{+}_{left}) - R(-v^{-}_{right} + v^{+}_{right}) + F_{p\_y} = 0 \tag{26}$$

In addition, at the plucking point, the left and right transverse string velocities must be continuous,

$$v^{-}_{left} + v^{+}_{right} = v^{-}_{right} + v^{+}_{right} \equiv v \tag{27}$$

where v is defined as the transverse velocity of the plucked point. Equations (26) and (27) allow us to solve for the outgoing velocity waves v^{-}_{left} and v^{+}_{right} in terms of the incoming velocity waves v^{-}_{right} and v^{+}_{left}:

$$\begin{cases} v^{-}_{left} = v^{-}_{right} + \dfrac{F_{p\_y}}{2R} \\ \\ v^{+}_{right} = v^{+}_{left} + \dfrac{F_{p\_y}}{2R} \end{cases} \tag{28}$$

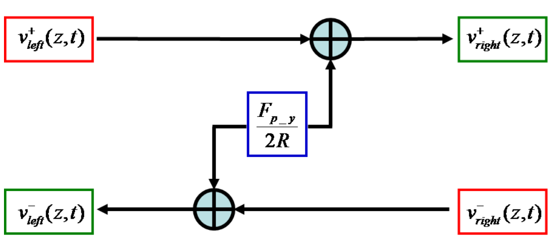

Figure 8: *Plectrum plucking junction.*

This plucking junction is shown in the diagram of Figure 8. The transverse displacement waves can be evaluated using a Backward Euler method:

$$\begin{cases} y^{-}_{left}(n) = y^{-}_{left}(n-1) + v^{-}_{left}(n)T \\ y^{+}_{right}(n) = y^{+}_{right}(n-1) + v^{+}_{right}(n)T \end{cases} \tag{29}$$

where T is the sampling interval. The transverse x displacement follows an identical derivation. When the string slides off the plectrum, $F_{p\_x} = F_{p\_x} = 0$, the plucking junction disappears, and the digital waveguide segments to the left and right of the junction are effectively combined into one.

5. RESULTS

5.1. Simulation Parameters

Table 1: *Delrin harpsichord plectrum and steel string values.*

Plectrum Parameters	
Length L	6 mm
Width W	4 mm
Thickness H	0.5 mm
Second moment of inertia I	0.029 mm^4
Young's modulus E	5 GPa
String Parameters	
Tension T	135 N
Density ρ	7850 kg/m^3
Diameter d	0.37 mm
Linear density μ	0.84 g/m
Length L_s	0.5 m

Modern harpsichord plectra are made out of a plastic material called Delrin. The plectrum and steel string parameters are listed in Table 1. The sampling frequency was chosen at $f_s = 100$ kHz. The plectrum width was made to equal that of one spatial sampling interval $X = 4.0$ mm. The harpsichord jack was assumed to move at a constant velocity v_j :

$$y_j(t) = v_j t$$

Referring to Figure 6 of the synthesis model, while the nut was treated as a rigid termination, we implemented a bridge filter that consisted of a one-pole filter and ripple filter similar to the one implemented in [28]. The transverse x and y string vibrations were not coupled together. That is a direction for future work.

5.2. String Excitation Motion

Figure 9 shows the transverse x and y string motion before the release of the string off the plectrum, plucked at the midpoint with a jack velocity $v_j = 0.02$ m/s. Clearly noticeable is a sharp steep rise in the horizontal displacement just prior to the release of the string that is absent in the vertical string displacement. This corresponds to the slip-off phase when the string is sliding off the plectrum tip. An expanded view of the slip-off portion is shown in Figure 10. This "kick" in the horizontal direction contributes to the brightness of the synthesized harpsichord tone.

5.3. Plucking Speed

Conventional wisdom has it that regardless of how fast one presses on the harpsichord key, the dynamics do not change considerably. Figure 11 shows a graph of the string release amplitude (defined as $A = \sqrt{x_s^2 + y_s^2}$ immediately before the release of the string from the plectrum) v.s. the jack velocity, plucked at the midpoint of the string. Under regular playing speeds of $0.02 - 0.1$ m/s, the

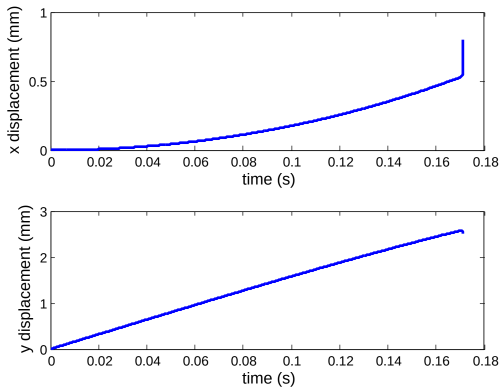

Figure 9: *Motion of string before release from plectrum, plucked at the midpoint with jack velocity v_j = 0.02 m/s.*

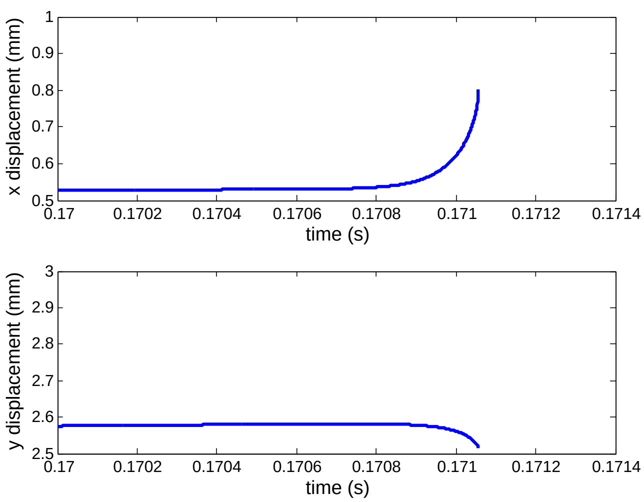

Figure 10: *Expanded view of the slip-off of of Figure 9.*

amplitude does not vary more than 10%, a difference not readily audible from our simulations. At higher playing speeds, there is a significant increase and drop in the release amplitude, but these are unphysical in the realm of harpsichord playing. In Figure 11 this "peak velocity" occurs at around 2 m/s. For longer bass strings, simulations show a lower the peak velocity but it remains well above reported playing speeds.

5.4. Plucking Point

Many harpsichords have more than one set of strings (called registers) for the same note, where the jacks pluck at different locations along the string. While Italian harpsichords generally had their plucking locations closer together for uniformity of sound, harpsichords built north of the Alps had their strings plucked at locations further apart to create differences in timbre [2]. It is well-established that plucking closer to the nut excites more harmonics

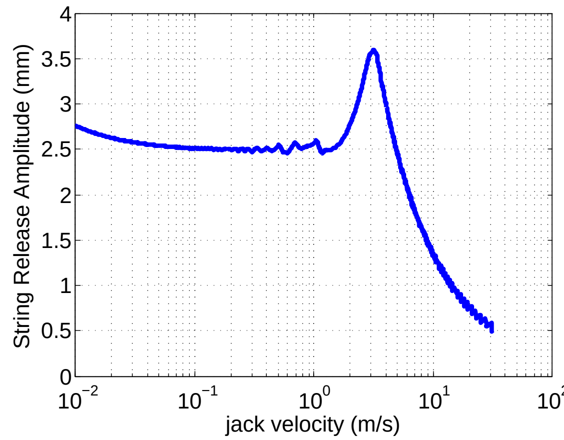

Figure 11: *String release amplitude v.s. jack velocity, plucked at the midpoint.*

and contributes to a nasal quality to the sound. Our simulations are consistent with this.

As discussed in [1], playing on harpsichord registers which pluck closer to the nut not only results in changes in timbre but also a decrease in volume. The string is released earlier, and the player experiences a "lighter" touch, as the harpsichord jack does not travel as far before the string is plucked. Figure 12 shows simulation results between the plucking location and string release amplitude. As expected, the largest amplitude occurs when plucked at the midpoint and decreases as the plucking point moves closer toward the nut.

6. CONCLUSION

This paper extends the previous harpsichord plectrum model proposed by the authors to incorporate the plectrum tip geometry. Interfacing with a digital waveguide, the complete plucked string motion, especially the final slip-off, is more accurately described. This is crucial in generating the string excitation signals to create realistic plucked harpsichord tones. Future work can include bridge coupling between the two transverse string vibrations and modeling of the lute stop.

7. ACKNOWLEDGMENTS

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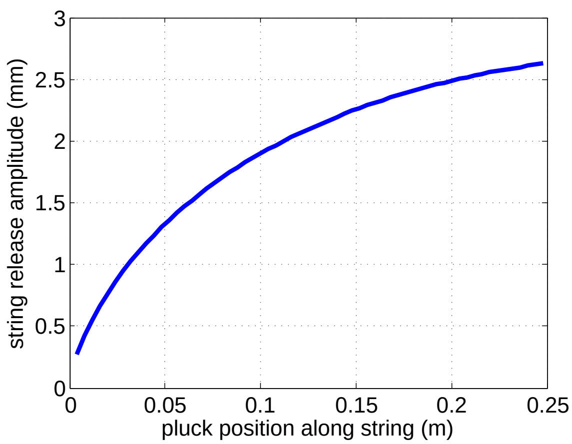

Figure 12: *String release amplitude v.s. plucking position, jack velocity* v_j = *0.02* m/s.

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