PHYSICAL PARAMETERS OF AN OSCILLATOR CHANGED BY ACTIVE CONTROL APPLICATION TO A XYLOPHONE BAR

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ABSTRACT

By applying active control to an oscillator, its modal behaviour changes. This paper makes a comparison between a second order damped harmonic oscillator and a xylophone bar's mode. Then it proposes a method for acting on an eigen resonance of a xylophone bar. The purpose is to get sound modifications, by bringing under quantitative and independent control its pitch and its duration. Thus it extends our previous work [1], by using a digital feedback controller.

1. INTRODUCTION

The sound emitted by a xylophone bar strongly depends on the localization and the shape of its eigen resonances. Thus changing the relative position and amplitude between the resonances should alter the bar's sound identity. This paper proposes a method for controlling in real time the resonance frequency and the damping factor of a xylophone bar mode. The variation of these parameters is intended to be independent and under quantitative control.

1.1. Active control of sound

Active control of sound is sound field modification. Various technological applications have been reported since the beginning of the last century, particularly around noise cancellation. The earliest patent, expressing how to compensate a sound signal by another one in antiphase, was submitted in 1933, by the German inventor P. Lueg [2]. The device is described by D. Guicking [3] and presented in Fig1.

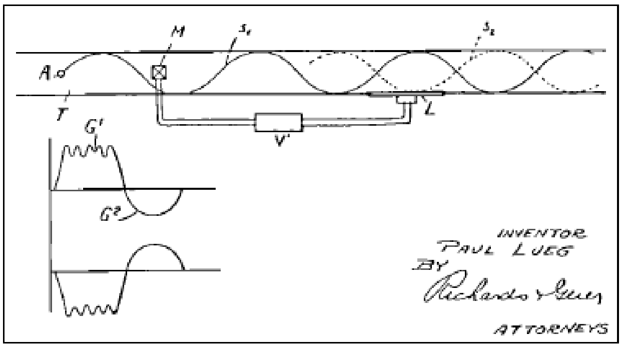

Figure 1: *Illustration page from P. Lueg's U.S. patent [2]*

In 1995, an active control method was applied to a xylophone bar in order to cancel the damping on a chosen vibration mode [1]. At the time we were using an analog feedback corrector which prevented us from achieving totally independent variations of the resonance frequency and the damping factor. The method described in this paper also uses an active control technique but with a digital feedback controller in order to get more accuracy on the mode parameters to modify.

1.2. Framework

The first part is dedicated to the theoretical review. First is described the behaviour of a second order harmonic oscillator when a PID (Proportional, Integral, Derivative) correction is applied. In the second part the transfer function of this system is compared to the frequency response of a xylophone bar around one of its eigen resonance. Then the validity of such a physical model will be discussed. Lastly a method is proposed for controlling in real time the resonance frequency and the damping factor of an eigen mode on the real xylophone bar.

2. PID CORRECTION ON A SECOND ORDER HARMONIC OSCILLATOR

The frequency response of a xylophone bar around one eigen resonance looks like the response of a second order damped harmonic oscillator. This assumption is verified below (cf part 3). This part describes the effect of a PID correction on the resonance frequency, the damping factor and the static gain of such an oscillator.

2.1. Expression of the oscillator's response parameters

Let us consider a {mass M, stiffness K, damping R} mechanical system as our theoretical model: cf Fig2

$$M\frac{du}{dt} + Ru + K\int u(t)dt = F_{ext} \quad (1)$$

where u is the velocity oscillation. The expression of the resonance pulsation is:

$$\omega_0 = \sqrt{\frac{K}{M}} \quad (2)$$

Assuming F_{ext} and u are the respective system input and output, equation (1) gives the transfer function in the Laplace space, plot-

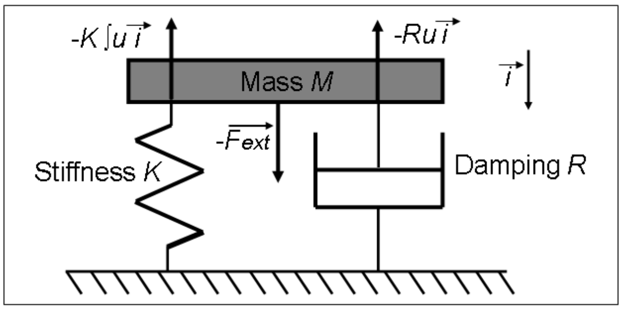

Figure 2: *System Mass, Stiffness, Damping: second order harmonic oscillator*

ted in Fig 3:

$$H(s) = \frac{U(s)}{F_{ext}(s)} = \frac{1}{Ms + R + \frac{K}{s}} \quad (3)$$

where s is the Laplace variable. The static gain H_0 is defined as the maximum value of $|H(j\omega)|$

$$H_0 = |H(j\omega_0)| = \frac{1}{R} \quad (4)$$

The damping factor Q indicates how selective the frequency response is; $Q = \frac{\omega_0}{\omega_2 - \omega_1}$, where $|H(j\omega_1)|_{dB} = |H(j\omega_2)|_{dB} = H_0|_{dB} - 3dB$. Therefore,

$$Q = \frac{\sqrt{KM}}{R} \quad (5)$$

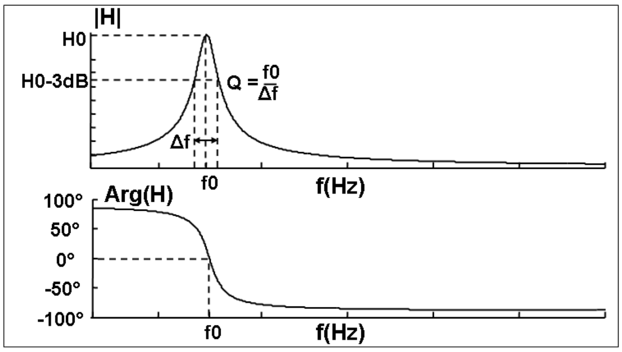

Figure 3: *Visualization of the oscillator's parameters: f_0, Q and H_0*

2.2. PID correction on the oscillator

A PID controller provides a force F_{corr}, which value is a sum of three terms proportional to its input u, its integral $\int u$, and its derivative $\dot{u}$:

$$F_{corr} = Pu + I\int u(t)dt + D\frac{du}{dt} \quad (6)$$

Then the relation (1) is modified:

$$(M - D)\frac{du}{dt} + (R - P)u + (K - I)\int u(t)dt = F_{ext} \quad (7)$$

(7) describes the motion of an other oscillator, whose parameters have been changed:

$$\omega_0' = \sqrt{\frac{K - I}{M - D}},\ Q' = \sqrt{\frac{(K - I)(M - D)}{R - P}},\ H_0' = \frac{1}{R - P} \quad (8)$$

While the variations of ω_0, Q and H_0 are defined by: $\Delta\omega_0 = \frac{\omega_0'}{\omega_0} - 1$, $\Delta Q = \frac{Q'}{Q} - 1$ and $\Delta H_0 = \frac{H_0'}{H_0} - 1$, the coefficients P, I and D can be expressed by:

$$P = \frac{\Delta H_0}{H_0(1 + \Delta H_0)} \quad (9)$$

$$I = \frac{Q\omega_0}{H_0}\left(1 - \frac{(1 + \Delta Q)(1 + \Delta\omega_0)}{1 + \Delta H_0}\right) \quad (10)$$

$$D = \frac{Q}{H_0\omega_0}\left(1 - \frac{(1 + \Delta Q)}{(1 + \Delta\omega_0)(1 + \Delta H_0)}\right) \quad (11)$$

Thus P, I and D are deduced from the desired variations $\Delta\omega_0$, ΔQ and ΔH_0, of the oscillator's parameters [4][5]. Fig 4 shows some examples of PID corrections applied to the described oscillator:

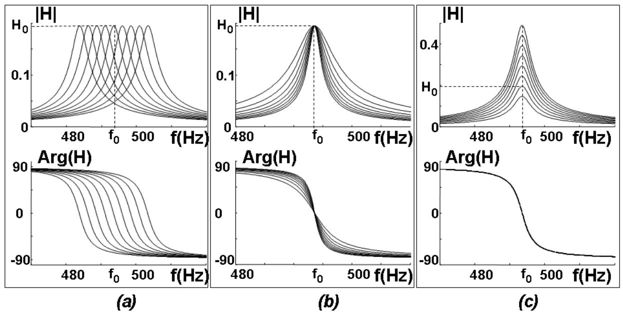

Figure 4: *(a)- PID correction of f_0: $-2\% \leq \Delta\omega_0 \leq 2\%$*
(b)- PID correction of Q:$-25\% \leq \Delta Q \leq 150\%$
(c)- PID correction of $H_0$$-25\% \leq \Delta H_0 \leq 150\%$

3. PID CORRECTION ON A REAL XYLOPHONE BAR RESONANCE

The purpose is to apply a PID correction to a real xylophone bar in order to modify one of its eigen modes. It would impose independent and quantitative control of its pitch, duration or sound intensity. On a real instrument some eigen modes may be coupled. In this case, modifying the parameters of a mode may imply non-linear variations on the next ones. However in a xylophone bar we assume that some eigen frequencies are isolated, far from the other. Thus a correction on one of them does not change the bar's frequency response much, in its vicinity. Here we chose to control the first eigen mode of the bar.

3.1. System description

It has been proved above that a PID correction is an effective method to control the movement of an oscillator. In the real case the correction force is applied to the xylophone bar thanks to an actuator. As this force should depend on the system's velocity, eq(6), a sensor is also required to catch the signal and to feed it to the controller. Thus the PID controller is integrated in a feedback loop: cf Fig5.

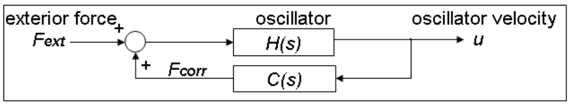

Figure 5: *Description of the looped system {oscillator + controller}*

3.2. Validity of the oscillator's model

Before applying a PID correction on a real instrument, the chosen model has to be checked. The xylophone bar's transfer function is measured around its first mode, and compared with the oscillator's response, cf Fig6. As the sensor and the actuator have been

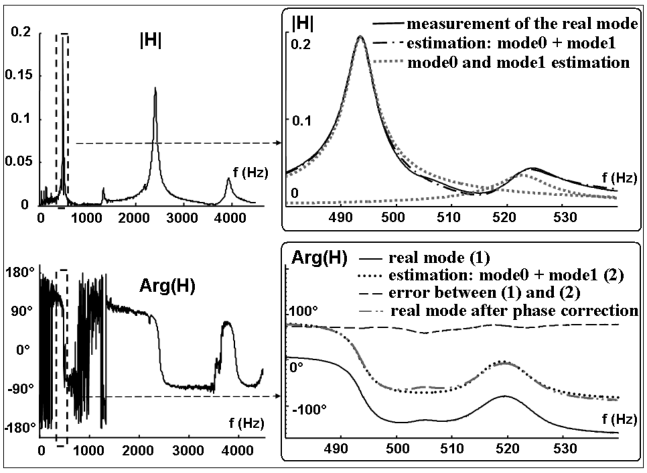

Figure 6: *Measurement of the real bar transfer function, zoom on the first mode*

linked to the bar, their physical effect on it is taken into account in the measured transfer function. Indeed the oscillator is a physical model of the whole system {actuator + xylophone bar + sensor}.The measured transfer function, Fig6, is composed by a sum of two very close resonances centred on 493Hz and 520Hz.Each of them is estimated by an algorithm which sums two harmonic oscillator's frequency responses, and compares the result with the measurement between 480Hz and 540Hz. The best estimations of both resonances have been plotted in Fig6. The modulus quadratic relative error (1.2%) is then minimal.

As a conclusion both measured system resonances are very similar to second order harmonic oscillator's responses. However the phase difference between estimation and measurement is larger and almost constant on the frequency band [480Hz, 540Hz]. This ought to be due to the fact that the sensor and the actuator are not colocalized.

3.3. Results of a PID correction

Three PID corrections are carried out on the first measured bar resonance (493Hz), cf Fig7. The PID controller is the theoretical one, which has been described in part 2.

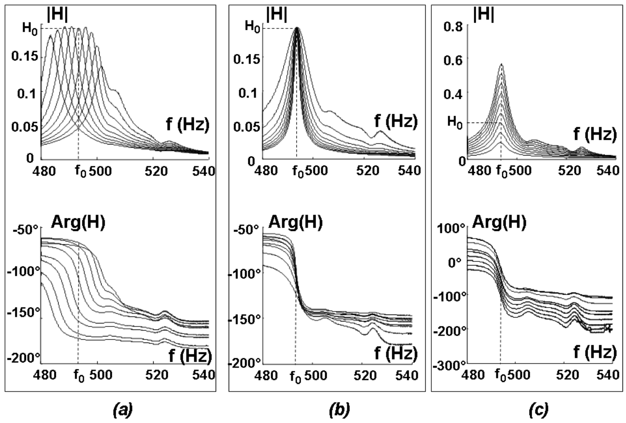

Figure 7: *PID correction, on the real transfer function, of:*
(a)- the of the resonance frequency: $-2\% \leq \Delta\omega_0 \leq 2\%$
(b)- the damping factor: $-25\% \leq \Delta Q \leq 150\%$
(c)- the static gain: $-25\% \leq \Delta H_0 \leq 150\%$

While ΔQ or $\Delta H_0 \in [-25\%, 150\%]$, these variations remain independent. However as soon as $\Delta\omega_0 < -0.5\%$ or $> 2\%$ this resonance pulsation modification comes with a bending of the mode's shape, it means an unwanted change of Q and H_0.

4. PID CORRECTION IN REAL TIME

Active control on an instrument becomes relevant as soon as its application is carried out in real time. This way, it enables the musician or the instrument maker to immediately modify the sound.

4.1. System description

In real time the feedback correction signal cannot be calculated instantaneously by the controller. That's why a variable delay is added into the loop so that the control signal is exactly 2π-dephased from the theoretical one. Moreover the PID controller is coded in C language and integrated into a digital signal processor (ARM microcontroller), in order to limit the introduced delay. Its input AD converter adds a very low offset in the signal that is fed to the PID controller. Then this offset is integrated so that the output DSP's level get saturated. That's why a digital high-pass filter $H(z) = \frac{G(1+z^{-1})}{1+\tau z^{-1}}$ is inserted in the controller just behind the integrator (Fig8). Its cut-off frequency is as low as possible ($60Hz$ here). Thus, since the PID transfer function has been modified, the relationship between P, I, D and $\Delta\omega_0$, ΔQ, ΔH_0 (eq(9),(10) and (11)) must be adapted too.

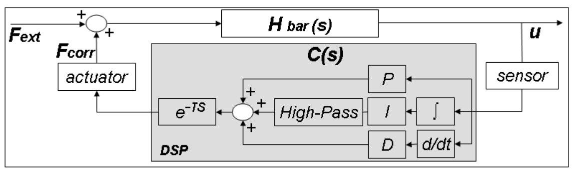

Figure 8: *Description of the real looped system {oscillator + real time controller}*

4.2. How to correct quantitatively, independently and in real time the parameters of the resonance

Let us assume that a second order harmonic oscillator is a valid model of the measured first mode, whose response is called H. The closed loop transfer function is a rational fraction of the Laplace variable s:

$$H_{tot}(s) = \frac{1}{\frac{1}{H(s)} - C(s)} \tag{12}$$

where C is the response of the PID controller containing the previous described integrator, cf Fig8. The method is intended to set up a one-to-one mapping table between each (P, I, D) and each (ω_{0c}, Q_c, H_{0c}), where ω_{0c}, Q_c, H_{0c} are the parameters of the target's transfer function. First in the expression of $|H_{tot}|$, ω is replaced by $\omega_{0c} + \Delta\omega$. It is then developed in power series around 0 ($\omega \approx \omega_{0c}$):

$$H_{tot}(s) = \sum_{k \in N} a_k \Delta\omega^k, when\ \Delta\omega \approx 0 \tag{13}$$

The maximum value of $|H_{tot}(s)|$ is intended to reach H_{0c} as soon as $\omega = \omega_{0c}$. So, when $\omega \approx \omega_0$:

$$\frac{\partial |H_{tot}|}{\partial \omega}|_{\omega=\omega_{0c}} = 0 \quad \Leftrightarrow \quad \sum_{k \in N*} k a_k \Delta\omega^{k-1} = 0 \tag{14}$$

$$\Rightarrow \quad a_1 = 0 \tag{15}$$

The $(a_i)_{i \in N}$ coefficients depend on the parameters of H (H_0,Q and ω_0) and of C (P, I, D and G, τ). Therefore (15) gives an expression of I in function of the other variables. This way, an algorithm tests a large range of P and D values, deduces I, and then compares the obtained response with the target. The best couple (P, D), which fits to the minimum relative quadratic error, is held in the table. The table is then completed by choosing another target mode.

4.3. Results

The described method is applied to the measured frequency response of the real xylophone bar. Various corrections are simulated by attempting to keep ω_0 constant (ie $f_{0c} = 493$Hz). The resulting responses plotted in Fig 9 are obtained by choosing random (P,D) values. Each of them have a different static gain (cf Fig9a) and a different damping factor (cf Fig9b), but the same resonance frequency.

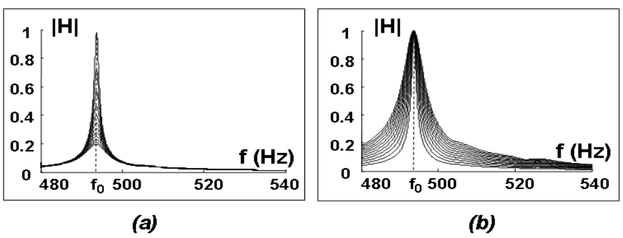

Figure 9:
(a)- Various corrections of Q and H_0 with ω_0 constant
(b)- Same corrections after normalization ($H_0 = 1$)

Thus the method enables to have H_0 and Q under quantitative control without modifying ω_0.

5. CONCLUSION

Acting on an acoustic instrument with an active control method is a way of diverting this technique from the traditional noise cancellation. Indeed by enhancing or damping an eigen mode, or by modifying its shape, we exploit larger possibilities than those usually used to soften vibrations. This new application of active control, in the music composition field, comes up to its expectation of new sounds, and reaches more and more fusion between the musician and its instrument [6].
The study described here is included in a larger work intended to build a generic method to control various sets of acoustic instruments by an active control technique. Finding a robust physical model of its resonances and an adapted corrector is a required stage for having them under digital and real time control. The very simple model described in part 2 is quite close to a real xylophone bar's mode. It shows that a PID correction is a possible theoretical way to modify an isolated xylophone resonance (part 3). By adapting it to real time requirements, some resonance modifications should be achieved as described in part 4. Some sounds come with these results. Now we still have the resonance frequency variations to bring under control by using the same digital corrector.
Once this method will be completed, it will be needed to considerate the effect of the coupling between modes in order to control the whole frequency response. Other works are in progress to apply the same technique to the violin bridge, by using another simple model described by Reinicke and Cremer [7], and by checking the results published by J. Woodhouse [8].

6. REFERENCES

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