DIGITAL SIMULATION OF THE DIODE RING MODULATOR FOR MUSICAL APPLICATIONS

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ABSTRACT

In this article, a model of the diode ring modulator is developed, which is based on a model from the literature, but modified to suit musical applications. After a brief introduction to analog ring modulation, a substitute circuit for the diode ring modulator is presented and analyzed, leading to a system of ordinary differential equations (ODEs) of first order. The equations are solved by using Euler's method. The model is compared with a real ring modulator using the same input waveforms, showing a good match between the simulation and the real device.

1. INTRODUCTION

An audio effect which can be found in many of today's digital audio workstations (DAWs) and synthesizers is the ring modulator. Originally used for transmission, ring modulators first became popular in music in the early 1970s, when Stockhausen based several of his works around the effect.

A ring modulator multiplies two waveforms, called *carrier* and *modulator* (Fig. 1). The resulting signal contains the sums and differences of the two signals' frequencies, also known as the upper and lower sideband. This follows directly from the trigonometric identity:

$$sin(f_M)sin(f_C) = \frac{1}{2}(cos(f_M + f_C) - cos(f_M - f_C)) \quad (1)$$

Analog ring modulation is typically implemented by using switches. The carrier waveform switches the polarity of the modulator. If the switching elements were ideal, this would only work for rectangular carrier waveforms. However in practice the switching elements (diodes, transistors) are not ideal. The V-I characteristic of silicon diodes, for instance, follows an exponential law. Thus, arbitrary carrier waveforms may be used. The resultant signal will however contain nonlinear distortions. In general the frequencies

$$f_O = |\pm nf_M \pm mf_C| \quad n, m = 0, 1, 2, 3... \quad (2)$$

may appear at the output. The even and odd order distortion products introduced are the main difference between analog ring modulation and a digital multiplication of two signals in the time domain.

Ring modulators are typically built either as active networks using transistors as switching elements, or as passive networks using diodes. This article will focus on the latter.

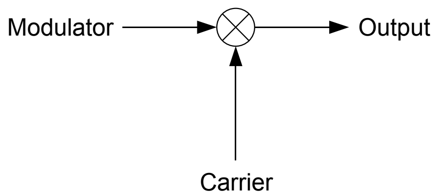

Figure 1: *Working principle of a ring modulator.*

2. MODEL OF THE RING MODULATOR

2.1. Circuit description

The diode ring modulator circuit is shown in Fig. 2. It consists of four diodes and two center-tapped transformers. The basic idea behind the circuit is that when the carrier voltage u_C is positive, diodes D_1 and D_2 are conductant while D_3 and D_4 block [1] [2]. When the carrier voltage u_C is negative, D_3 and D_4 are conductant while D_1 and D_2 block. This is due to the carrier, which is connected to the center taps of both transformers — the same carrier current flows through both diode pairs. Additionally this causes the carrier currents to cancel out each other at the second transformer, leaving only the desired modulation product at the output.

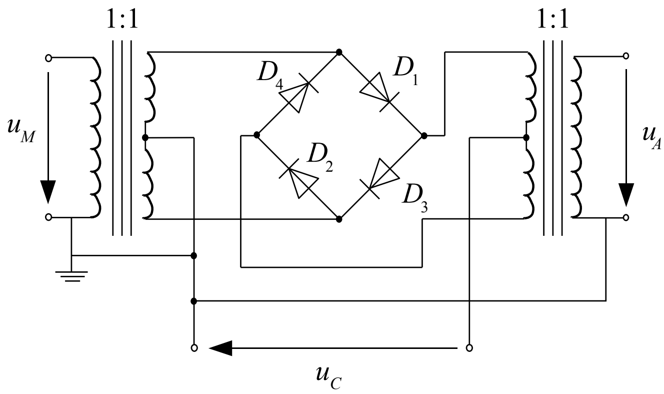

Figure 2: *Ring modulator circuit.*

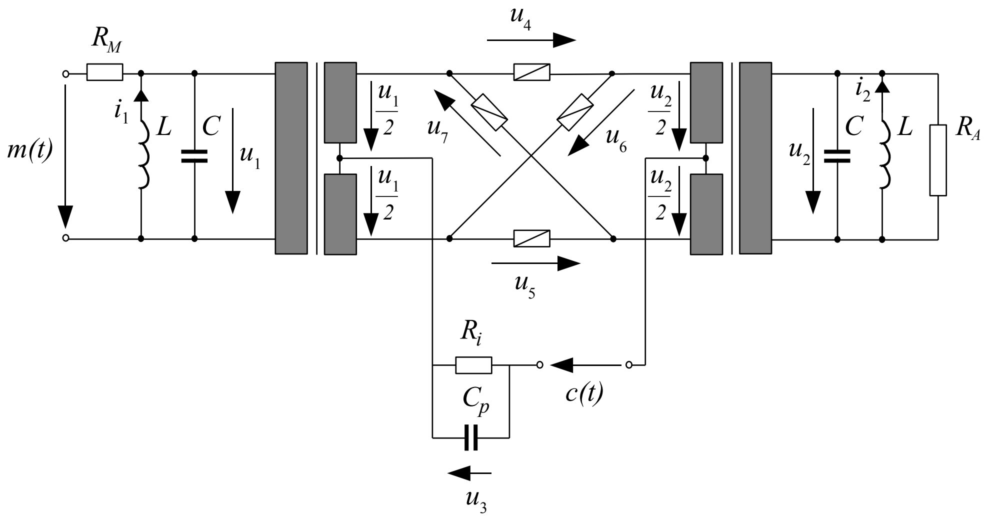

Figure 3: *Substitute circuit for the diode ring modulator.*

2.2. Substitute circuit and analysis

The model proposed in this paper is a modified version of the Horneber model [3]. The original design leads to 15 differential equations, mainly due to the transformer model used. The simplified substitute circuit suggested here is given in Fig. 3. It consists of ideal transformers connected in parallel with a capacitance C and inductance L. Together, they form a nonideal transformer. The four diodes are replaced by voltage controlled resistors. The capacitance C_p is used to regularize the model. R_M and R_i are source resistances, R_A is the load resistance. From Kirchhoff's current law (KCL) a system of 5 ordinary differential equations of first order is obtained:

$$C\frac{du_1}{dt} = i_1 - \frac{g(u_4)}{2} + \frac{g(u_7)}{2} + \frac{g(u_5)}{2} - \frac{g(u_6)}{2} - \frac{u_1 - m(t)}{R_M} \quad (3)$$

$$C\frac{du_2}{dt} = i_2 + \frac{g(u_4)}{2} - \frac{g(u_6)}{2} - \frac{g(u_5)}{2} + \frac{g(u_7)}{2} - \frac{u_2}{R_A} \quad (4)$$

$$C_p\frac{du_3}{dt} = g(u_4) + g(u_5) - g(u_6) - g(u_7) - \frac{u_3}{R_i} \quad (5)$$

$$L\frac{di_1}{dt} = -u_1 \quad (6)$$

$$L\frac{di_2}{dt} = -u_2 \quad (7)$$

The voltages across the diodes can be derived from Kirchhoff's voltage law (KVL):

$$u_4 = \frac{u_1}{2} - u_3 - c(t) - \frac{u_2}{2} \quad (8)$$

$$u_5 = -\frac{u_1}{2} - u_3 - c(t) + \frac{u_2}{2} \quad (9)$$

$$u_6 = \frac{u_1}{2} + u_3 + c(t) + \frac{u_2}{2} \quad (10)$$

$$u_7 = -\frac{u_1}{2} + u_3 + c(t) - \frac{u_2}{2} \quad (11)$$

Technical parameters are $C = 10e^{-9}F$, $C_p = 10e^{-9}F$, $L = 0.8H$, $R_A = 600\Omega$, $R_i = 50\Omega$ and $R_M = 80\Omega$. The diode function g is a nonlinear function of the voltages across the diodes, hence the system is nonlinear. The inputs are given by the carrier signal $c(t)$ and the modulator signal $m(t)$. The output signal is determined by u_2.

The original model by Horneber also simulates the leakage flux in the transformers. However, this affects only high frequencies [4]. Measurements on audio transformers indicate that they fall outside the audible range and thus need not be modelled. Another issue is that additional artificial capacitances are needed to obtain explicit differential equations, greatly increasing the numerical problems when attempting to simulate the design [5] [6] [7].

2.3. Diode model

A suitable function g is required to model the diode behavior. Real ring modulators use germanium diodes due to their soft slopes and fast turn-on. The V-I characteristic of germanium diodes, when biased forward at a small voltage, can be well approximated by a polynomial. At reverse bias, the voltage may be set to zero. The following equation approximates a 1N270 germanium diode at low voltages (x is given in Volt, $g(x)$ in Ampere):

$$g(x) = \begin{cases} 0.17x^4 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$$

Using silicon diode models will not work. Since the bias point of the diodes is at $0V$, this causes the carrier waveform to 'squash' the modulator waveform near its zero crossings, creating harsh distortions at the output. It is possible, however, to shift the bias point of the diodes to a steeper region of the exponential curve. This is simply accomplished by adding a constant to the voltage applied to the diode. Using the model from [3] for a 1N4448 silicon diode and adding a constant voltage of $300mV$ results in

$$g(x) = 40.67286402e^{-9}(e^{17.7493332(x+0.3)} - 1)$$

Both diode models yield good results in practice. Fig. 4 shows the V-I diagram of both functions.

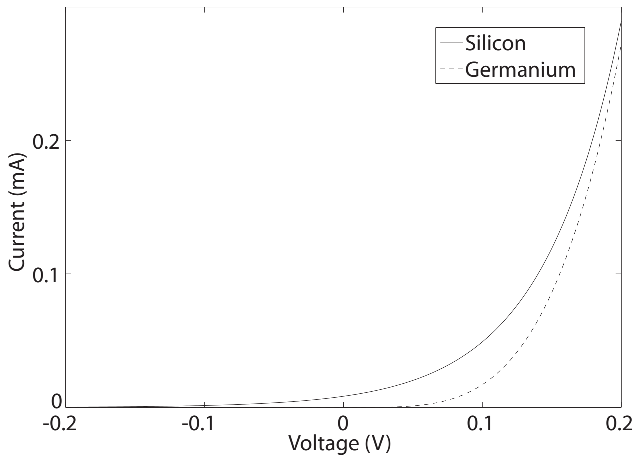

Figure 4: *Diode functions.*

2.4. Transformer model

The inductances L affect the frequency response of the circuit, acting like a level-dependent high-pass filter. Figure 5 shows the frequency response of the circuit, setting the carrier to DC at two different levels. Smaller values of L increase the high-pass characteristic, larger values decrease it. If a linear frequency response is desired, Eq. 6 and 7 may be removed from the ODE set entirely and only 3 nonlinear equations need to be solved.

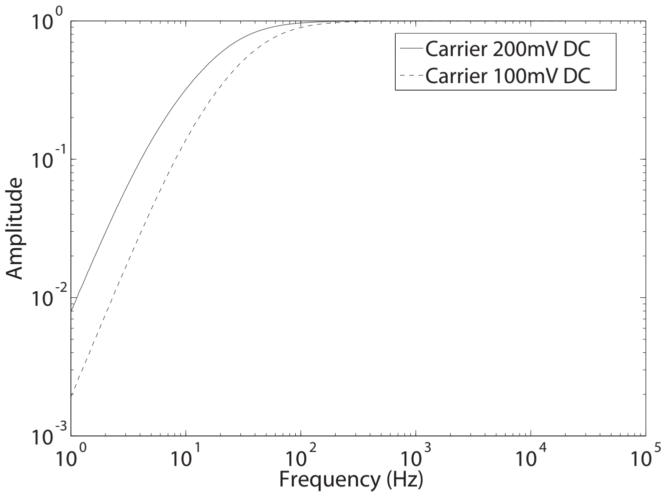

Figure 5: *Frequency response at different carrier levels.*

3. DISCRETIZING THE MODEL

In order to use the ring modulator model, it must be discretized. Several methods for discretizing ODEs used for audio processing have been recently discussed in [8]. The easiest way to solve an ODE is the Forward Euler method. Applying it to Eq. 3 to 7 yields

$$u_1(n) = u_1(n-1) + \frac{T}{C}(i_1(n-1) - \frac{g(u_4)}{2} + \frac{g(u_7)}{2} + \frac{g(u_5)}{2} - \frac{g(u_6)}{2} - \frac{u_1(n-1) - m(n)}{R_M}) \quad (12)$$

$$u_2(n) = u_2(n-1) + \frac{T}{C}(i_2(n-1) + \frac{g(u_4)}{2} - \frac{g(u_6)}{2} - \frac{g(u_5)}{2} + \frac{g(u_7)}{2} - \frac{u_2(n-1)}{R_A}) \quad (13)$$

$$u_3(n) = u_3(n-1) + \frac{T}{C_p}(g(u_4) + g(u_5) - g(u_6) - g(u_7) - \frac{u_3(n-1)}{R_i}) \quad (14)$$

$$i_1(n) = i_1(n-1) + \frac{T}{L}(-u_1(n-1)) \quad (15)$$

$$i_2(n) = i_2(n-1) + \frac{T}{L}(-u_2(n-1)) \quad (16)$$

where T is the time interval between samples. Eq. 8-11 remain the same, except that $c(t)$ is replaced by $c(n)$. The levels of the modulator and carrier signals should be within $(-200mV, +200mV)$. Since the system is stiff, any explicit method such as Forward Euler requires oversampling to ensure numerical stability. The oversampling factor required depends on the input signals. In general, higher signal levels will require a higher oversampling factor. If a low oversampling factor with arbitrary signals is desired, implicit or semi-implicit methods [9], which possess better numerical stability, must be used to solve the ODEs. However, they are not as easy to implement as explicit methods.

4. RESULTS AND DISCUSSION

In order to compare the proposed model with a real ring modulator, an electric bass sound was modulated by a sinusoidal carrier waveform of 128 Hz. The levels were chosen such that the nonlinear distortions were audible, yet not objectionable. The Forward Euler method with an oversampling factor of 128 was used to solve the ODEs. The analog ring modulator used consisted of four 1N270 germanium diodes and two Mouser TM018 transformers. Fig. 6 shows the time-domain output of the real diode ring modulator. The output of the model is presented in Fig. 7, showing a good match. An even better match would be possible by optimizing the model parameters to fit the device under test. However since real ring modulators show an asymmetric behavior (diodes and transformer coils never match exactly, causing a fraction of the carrier and modulator signals to appear at the output), a perfect match is not necessarily desirable.

5. ACKNOWLEDGMENTS

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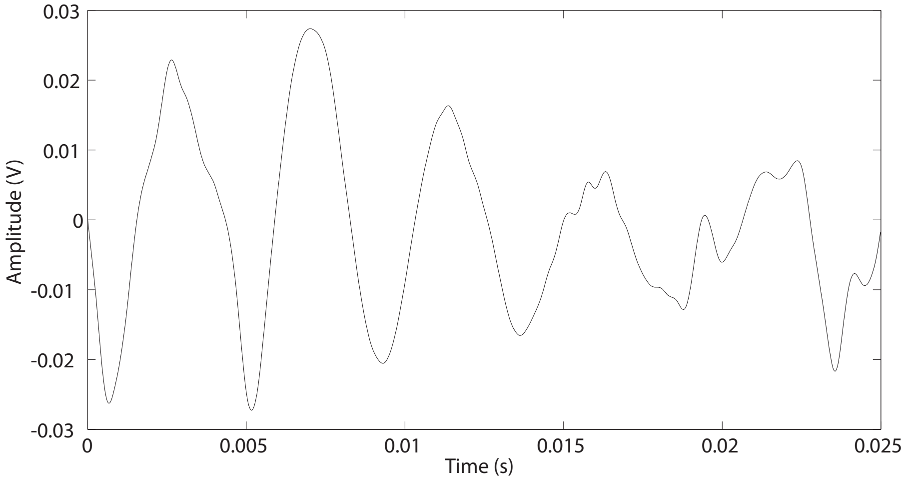

Figure 6: *Output voltage measured on a real ring modulator.*

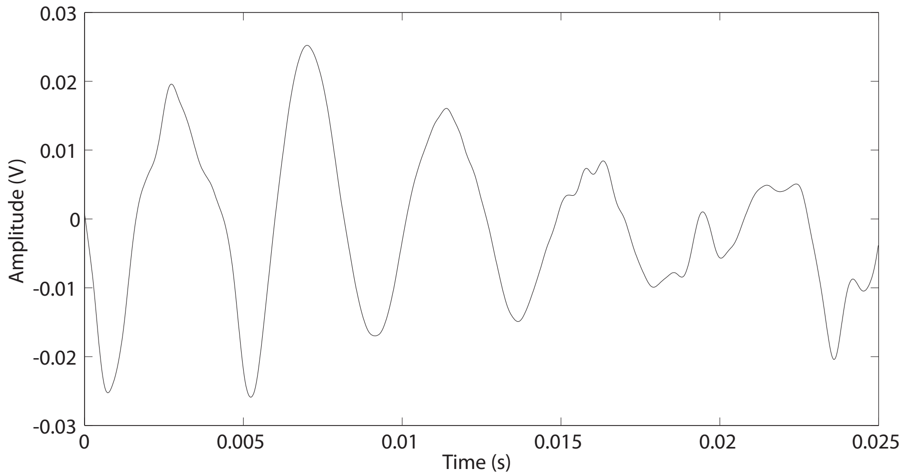

Figure 7: *Output voltage from the model.*

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