COEFFICIENT-MODULATED FIRST-ORDER ALLPASS FILTER AS DISTORTION EFFECT

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ABSTRACT

A novel approach to implement the distortion effect is introduced. The proposed approach is based on time-varying phase distortion of the input signal, and it is implemented using a coefficient modulated first-order allpass filter. This new technique provides control over the distorted band with a proper choice of the modulating signal. By choosing a modulating signal that applies the phase distortion only for low frequencies, the aliasing often generated by conventional distortion effects, which modify the signal amplitude, can be greatly reduced. Modulation signals that produce distortion effects applicable for electric guitar playing are also discussed. Sound examples on the use of the filter can be found at http://www.acoustics.hut.fi/~jpekonen/Papers/dafx08/.

1. INTRODUCTION

Distortion, a controversial property of an electrical device, is desired to be minimized in high fidelity sound reproduction systems, but in music, and especially in electric guitar playing, the distortion effect is an essential tool in the generation of new timbres and soundscapes. Examples on the use of the distortion applied to guitar playing can be heard in any modern day rock album as the effect has become a standard part of the sound.

The operation principle of the distortion effect is to modify an input signal $x(n)$ with a nonlinear function $f(\cdot)$, i.e.,

$$y(n) = f(x(n)). \quad (1)$$

The distortion function $f(\cdot)$ introduces to the output $y(n)$ new frequency components that are not present in the input signal, and it is usually implemented either as a table or as a polynomial approximating the characteristics of an analog nonlinear circuit. Quite often the distortion function is a hyperbolic trigonometric function, e.g. the hyperbolic sine $\sinh$ or the hyperbolic tangent $\tanh$, a nonlinearity often found in electric circuits [1, 2], or a sum of Chebychev polynomials [3], which produce the harmonic component of their order to a sinusoidal input.

The above-mentioned approaches can also be used in sound synthesis, as in the waveshaping synthesis technique the sound is produced by applying a nonlinear function to a sinusoidal input. The use of the sinusoidal input signal in the waveshaping synthesis is due to the fact that the distortion obtained by (1) produces aliasing when either the distortion function $f(\cdot)$ contains any discontinuities or the input signal has a complex waveform, i.e., it contains energy also at high frequencies [4, 5].

The distortion implemented as (1) modifies the amplitude of a signal. Since an arbitrary signal can be represented at any time instant using two properties, namely amplitude and phase, the nonlinear amplitude modification can also be interpreted as a modification of the phase increment of the input signal from one sample to another. Therefore, the distortion effect obtained by (1) could be implemented by modifying the phase of the input signal instead of the amplitude. This point of view is illustrated in Figure 1, where the output of a distorter $f(x) = \tanh(2x)/\tanh(2)$ applied to one cycle of a sinusoid is presented. The factor $1/\tanh(2)$ is used to set the maximum amplitude of the output to unity in order to illustrate this viewpoint more effectively.

Figure 1 shows that first the amplitude of the distorted sinusoid increases faster than that of the pure sine, which can be obtained by increasing the phase increment of the sampling synthesizer. When the amplitude of the output is about to reach unity, the phase increment is decreased so that it is almost zero. After the maximum, the phase increment is modified as before the maximum but in a time reversed manner. Since the $\tanh$ function is symmetric, the negative part of the output can be obtained by similar modifications of the phase increment as for the positive part.

Phase distortion has been previously used for sound synthesis purposes, but those implementations apply the phase distortion only to waveforms read from a table, e.g. see [6]. For an arbitrary signal the phase distortion can be implemented by means of adaptive frequency modulation (AdFM) [7], where the input signal is fed into a delay line from which the output is read in the desired phase increment. However, in order to produce modifications that correspond exactly to a conventional amplitude distorting function, the AdFM approach would require a control logic that would extract the modulation parameters from the amplitude of the input signal. Since the mapping between the amplitude and the modulation parameters is, in principle, the same as the implementation of the conventional amplitude distortion function, the direct AdFM approach is not practical.

The computational complexity of the AdFM could be reduced by simplifying the control parameters to be constant or directly obtainable, e.g. via filtering, from the input signal. However, since in a system with limited computational capabilities both the number of required operations and the number of required delay elements are desired to be as small as possible, the delay line required in AdFM needs to be replaced with a filter structure that utilizes a smaller number of delay elements. Since a delay line is, in principle, an allpass filter, i.e. the magnitude response of a delay is unity for all frequencies, a low-order allpass filter can be used to approximate the delay line.

This paper introduces a novel approach to implement the distortion effect via a time-varying, coefficient-modulated first-order allpass filter. In Section 2, the structure of the filter and its effect on an input signal are presented. Section 3 discusses the prop-

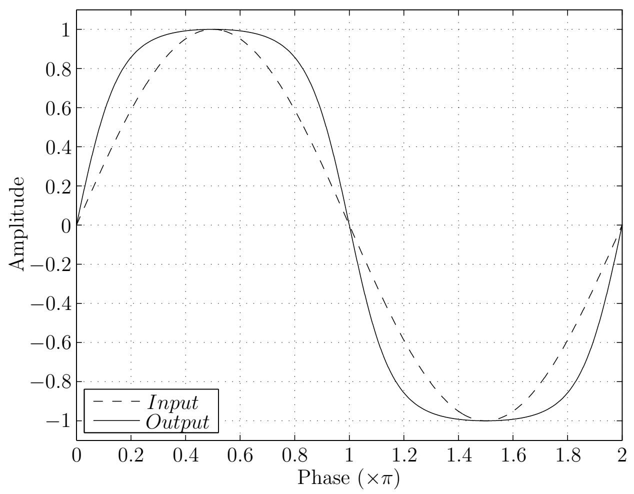


Figure 1: The output (solid line) of a distorter $f(x) = \tanh(2x)/\tanh(2)$ applied to a sinusoid (dashed line).

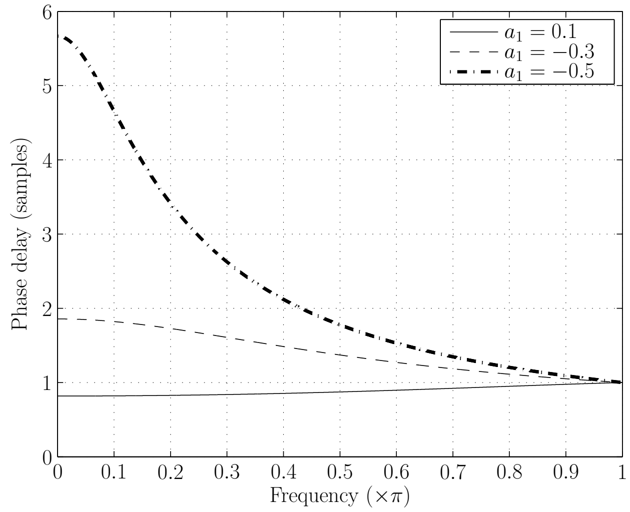


Figure 2: The phase delay of a first-order allpass filter with a coefficient value of 0.1 (solid line), −0.3 (dashed line) and −0.5 (dash-dotted line).

erties of the presented implementation with stability analysis and with the choice of the phase distortion signal. The use of the time-varying filter is discussed in Section 4 for the distortion type used in electric guitar playing. Section 5 concludes the paper.

2. COEFFICIENT-MODULATED FIRST-ORDER ALLPASS FILTER

By definition, a time-invariant allpass filter passes the input signal without affecting its magnitude spectrum. Instead, allpass filters modify the phase of the signal, thus implementing a frequency-dependent delay determined by the filter coefficients. This is illustrated in Figure 2 where the phase delay, the delay applied by a filter to an individual sinusoid [8], of a first-order allpass filter,

$$H(z) = \frac{a_1 + z^{-1}}{1 + a_1 z^{-1}}, \tag{2}$$

is plotted for three values of the coefficient a_1. As can be seen in Figure 2, the phase delay varies as the coefficient a_1 is varied, more at low frequencies than at high. At DC, the phase delay is given by [9]

$$D_{\mathrm{DC}} = \frac{1 - a_1}{1 + a_1}. \tag{3}$$

At the Nyquist limit, the phase delay is always exactly the filter order, regardless of the coefficient value.

The variation of the phase delay can be interpreted as a distortion effect as discussed above. By allowing the coefficient a_1 of the first-order filter to be time-varying according to a modulating signal $m(n)$, the filter produces a time-varying frequency-dependent delay which can be interpreted as a modification of the phase increment of an input signal from one sample to another. Now, the resulting filter is no longer allpass and the magnitude response of the filter depends on the modulating signal $m(n)$.

The flow diagram of a coefficient modulated allpass filter is given in Figure 3 and the difference equations for the filter state and output are given by

$$\begin{cases} w(n) = x(n) + m(n)y(n) & \text{and} \\ y(n) = -m(n)x(n) + w(n-1), \end{cases} \tag{4}$$

where $w(n)$ is the resulting signal of the first adder fed to the delay element as shown in Figure 3. The state formula, i.e., the formula for $w(n)$ in (4), can be expanded as

$$\begin{aligned} w(n) &= (1 - m^2(n))x(n) \\ &\quad + \sum_{k=1}^{\infty}\prod_{l=0}^{k-1} m(n-l)(1 - m^2(n-k))x(n-k). \end{aligned} \tag{5}$$

Now, the output of the filter can be written as

$$\begin{aligned} y(n) &= -m(n)x(n) + (1 - m^2(n-1))x(n-1) \\ &\quad + \sum_{k=2}^{\infty}\prod_{l=1}^{k-1} m(n-l)(1 - m^2(n-k))x(n-k). \end{aligned} \tag{6}$$

If the filter is assumed to be causal, the summation goes to n instead of infinity. With (6), the effect applied by the filter can be expressed in closed form when both the input $x(n)$ and the modulation signal $m(n)$ are known.

A time-varying first-order allpass filter has been previously used to model nonlinear behavior of acoustical instruments, e.g. a nonlinear spring termination [10] and the tension modulation phenomenon [11, 12] of a vibrating string. In the tension modulation model the time-varying first-order allpass filter is used to implement a fractional delay, i.e. the phase delay of the filter at DC is between zero and one, and in the nonlinear spring termination model the filter coefficient is switched between two values. However, here the number of possible modulation signal values is not limited and the DC phase delay can be greater than one.

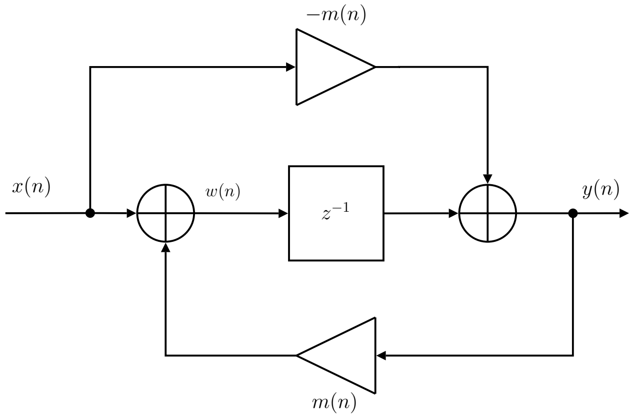


Figure 3: Flow diagram of the coefficient-modulated first-order allpass filter.

3. PROPERTIES OF THE COEFFICIENT-MODULATED FIRST-ORDER ALLPASS FILTER

First, the stability condition of the presented filter must be determined. A thorough discussion on the stability analysis of time-varying filters can be found in [13], and the stability analysis of the proposed filter is performed here using the state-space stability analysis approach. For an arbitrary state-space representation given by

$$\begin{cases} X(n+1) = P(n)X(n) + Q(n)x(n) \\ y(n) = R(n)X(n) + S(n)x(n), \end{cases} \tag{7}$$

where $x(n)$ and $y(n)$ are the input and output signals, respectively, $X(n)$ is the state vector of the filter, and $P(n)$, $Q(n)$, $R(n)$ and $S(n)$ are filter-dependent time-varying transition coefficients, the necessary and sufficient condition for time-varying bounded-input-bounded-output (BIBO) stability can be expressed for all n with [13]

$$|S(n)| + \sum_{i=n-1}^{-\infty} \left| R(n) \left(\prod_{k=i+1}^{n-1} P(k) \right) Q(i) \right| < G, \tag{8}$$

where G is bounded.

For the proposed filter the state-space representation coefficients are $X(n+1) = w(n)$, $P(n) = m(n)$, $Q(n) = 1 - m^2(n)$, $R(n) = 1$ and $S(n) = -m(n)$. By applying the triangle inequality to the sum term, an upper bound for the BIBO condition can be obtained,

$$\begin{aligned} &|m(n)| + \sum_{i=n-1}^{-\infty} \left| (1 - m^2(i)) \left(\prod_{k=i+1}^{n-1} m(k) \right) \right| \\ \leq\ &|m(n)| + \sum_{i=n-1}^{-\infty} \left(|1 - m^2(i)| \prod_{k=i+1}^{n-1} |m(k)| \right). \end{aligned} \tag{9}$$

This upper bound converges if $|m(k)| < 1$ for all k since then the product term converges towards zero as i decreases. In addition, if $|m(k)| = 1$, the sum term reduces to zero for all n. Therefore, the filter is stable if $|m(n)| \leq 1$ for all n. The same condition for the filter stability can be obtained by writing the impulse response of the filter using (6) and by analyzing the convergence of the absolute sum of the impulse response samples.

The delay produced by the coefficient modulated allpass filter at zero frequency at time instant n can be computed using (3). Since now $a_1 = m(n)$, the DC delay at time instant n is then

$$D(n) = \frac{1 - m(n)}{1 + m(n)}. \tag{10}$$

Since the filter stability requires that $|m(n)| \leq 1$ for all n, the DC delay created by the filter is always nonnegative. This implies that the effect illustrated in Figure 1 cannot be achieved with the proposed filter, as going forwards in the signal phase would require a negative delay.

Consider now the extreme values of the modulating signal $m(n)$. When $m(n) = 1$, the DC delay created is zero, and it is infinite when $m(n) = -1$. When the DC delay is large, the delay at low frequencies is also quite large, and the filter is very dispersive, i.e. the high frequencies exit the filter well before the low frequencies. When the input signal contains energy also at high frequencies, the dispersion effect would produce unnatural artifacts that are not desirable in the distortion effect. Therefore, the modulating signal values should not be close to -1 for long times.

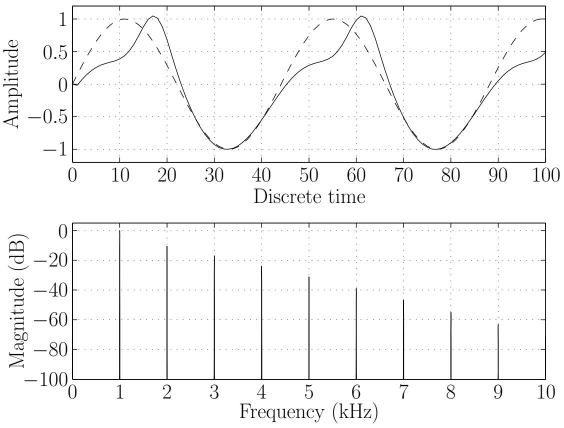
Another issue in time-varying filters is the transients induced by the coefficient changes. In many applications these transients are not desirable, and techniques to remove them have been proposed, see e.g. [14, 15]. However, as the transients produce distortion, they can be tolerated in the distortion effect. Yet, the coefficient change may produce a DC shift to the output signal, and in order to avoid numerical overflows the DC shift can be removed by placing an additional scaling factor on either side of the delay of the filter [16]. However, in the proposed filter structure this scaling factor has been omitted.

The distortion effect obtained by the filter is illustrated in Figure 4, where three modulation signal examples for an individual sinusoidal input signal are given. When the modulating signal gets both positive and negative values, the filter applies a larger distortion than when the modulation signal gets only values of one sign. In addition, when the modulation signal is positive, the distortion obtained is larger than that obtained with a negative modulation signal. In Figure 5, the same modulation signal examples are given for a sinusoidal input of another frequency.

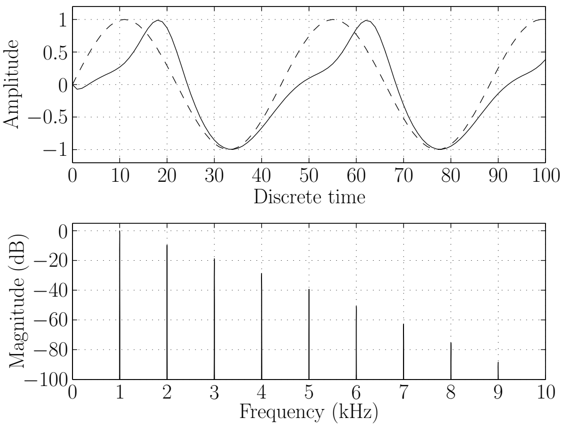
The examples given in Figures 4 and 5 illustrate another aspect of the operation of the proposed filter. With the choice of the modulating signal different frequencies of the input signals are modified differently, and the effect acts like a selective frequency modulator. The low frequencies are affected the most with all modulation signals, but when $m(n)$ gets only positive values high frequencies are affected more than when $m(n)$ gets only negative values. When the input signal contains energy also at high frequencies, the conventional distortion effects modifying the signal amplitude would suffer from aliasing if no oversampling is used. Using the coefficient modulated allpass filter that applies the effect only at low frequencies the aliasing issue can be reduced.

4. MODULATION SIGNAL CHOICE FOR ELECTRIC GUITAR PLAYING

The distortion effects used in electric guitar playing vary a lot both with respect to the amount of distortion applied and the describing



(a)



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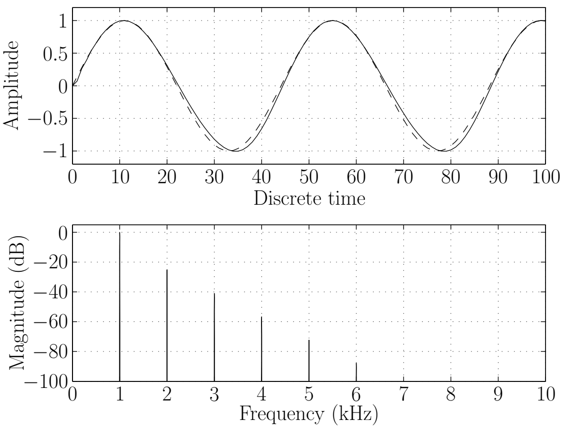
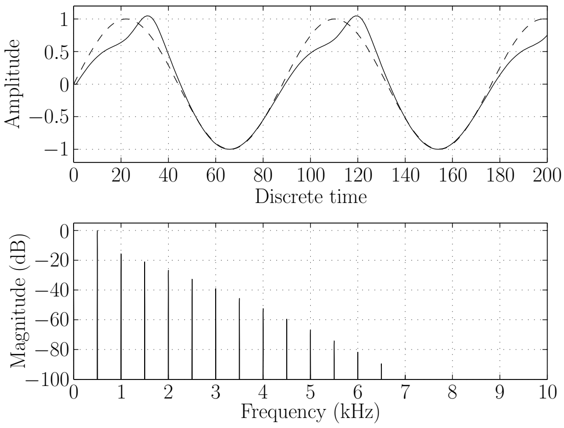
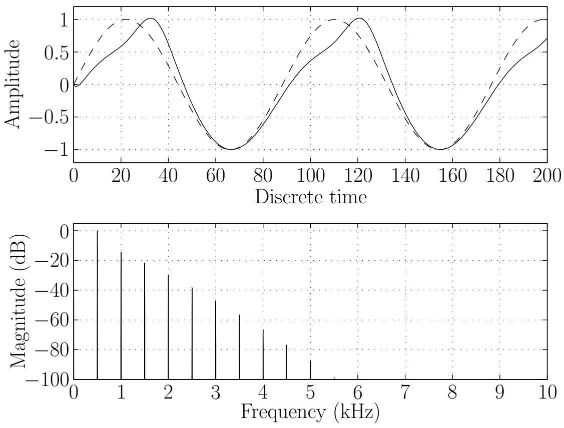


(c)

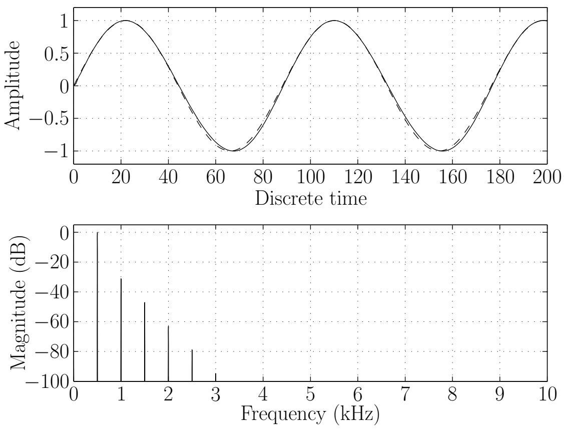
Figure 4: The output of the effect applied by the coefficient-modulated first-order allpass filter to a 1000 Hz sinusoid in time- and frequency-domains obtained by using (a) $m(n) = 0.9x(n)$, (b) $m(n) = 0.45+0.45x(n)$, and (c) $m(n) = -0.45-0.45x(n)$. The sampling frequency used is 44.1 kHz.



(a)



(b)



(c)

Figure 5: The output of the effect applied by the coefficient-modulated first-order allpass filter to a 500 Hz sinusoid in time- and frequency-domains obtained by using (a) $m(n) = 0.9x(n)$, (b) $m(n) = 0.45+0.45x(n)$, and (c) $m(n) = -0.45-0.45x(n)$. The sampling frequency used is 44.1 kHz.

names. One may call a certain type of distortion "crunch" while someone else may use that name for a different distortion type. Here, some practical choices of the modulation signal $m(n)$ for different distortion effects are discussed without using any names for the distortion effects.

When only a mild distortion is desired, the amplitude of the signal is subject to be modified lightly, whereas for a heavier distortion the amplitude modifications are more drastic. These amplitude modifications map to phase distortions similarily; for a larger distortion a larger phase modification and vice versa. In addition, the desired type of distortion is also affected by the range of values where the modulation signal $m(n)$ is varied as the range defines how the phase distortion is applied to different frequencies. This aspect was illustrated in Figures 4 and 5.

However, one question still remains open: What kind of modulation signal must be used to obtain a proper distortion effect? One could use the input signal to drive the modulation, but, since in many cases the input signal is non-smooth, thus producing faster phase distortion variations and hence larger distortion, the input signal should be lowpass filtered in order to smoothen the phase distortion variations. Yet, one could use a constant modulation signal, e.g. a sinusoid of a certain frequency, thus reducing the complexity of the modulation signal retrieval. However, a constant modulation signal would produce slightly different distortion effects for different input signals, which is in conflict with the output obtained by the conventional distortion effects. Yet, the resulting signal can be interesting, and some sound examples of such cases can be found at `http://www.acoustics.hut.fi/~jpekonen/Papers/dafx08/`.

5. CONCLUSIONS

A novel approach to implement a distortion effect was introduced. The proposed approach was based on time-varying phase distortion of the input signal, and its implementation as a coefficient-modulated first-order allpass filter was presented. The properties of the presented implementation were discussed, and the property of being a selective frequency modulator was illustrated. Practical guidelines for the choice of the modulation signal for different types of distortion were also discussed.

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