THE BEATING EQUALIZER AND ITS APPLICATION TO THE SYNTHESIS AND MODIFICATION OF PIANO TONES

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ABSTRACT

This paper presents an improved method for simulating and modifying the beating effect in piano tones. The beating effect is an audible phenomenon, which is characteristic to the piano, and, hence, it should be accounted for in realistic piano synthesis. The proposed method, which is independent of the synthesis technique, contains a cascade of second-order equalizing filters, where each filter produces the beating effect for a single partial by modulating the peak gain. Moreover, the method offers a way to control the beating frequency and the beating depth, and it can be used to modify the beating envelope in existing tones. The results show that the proposed method is able to simulate the desired beating effect.

1. INTRODUCTION

The beating effect is one of the audible characteristics in piano tones [1]. It occurs due to the coupling of detuned strings. Even if there is only one string per a key, as in the first keys of the piano, beating can be present due to false coupling [2]. As the beating effect is a perceptually important phenomenon, it must be taken into account in a realistic piano synthesis model.

Various beating effect simulations have been proposed for digital waveguide synthesis. In the first waveguide models the beating effect was produced with parallel detuned string models [3, 4]. Bank suggested a resonator-based approach, where a resonator is tuned close to the frequency of the target partial, which produces the beating effect due to frequency modulation [5, 6]. In addition, a multi-rate version of the resonator-based approach has been proposed [7]. In the resonator-based approach, the frequency of the partial must be known in order to control the beating frequency. Moreover, the approach does not provide straight-forward control over the depth of beating. Additionally, Bank and Sujbert have suggested a method using pitch-shift to produce the beating effect [8].

Rauhala et al. [9] proposed a beating model, where the beating effect is, first, simulated by separating the partial from the signal with a bandpass filter. Then, the partial is modulated with a low-frequency oscillator (LFO). Finally, the modulated partial is added to the original signal. This approach does not require exact knowledge of the frequency of the partial and it provides an easy way to control the depth of the beating. On the other hand, the depth control is not very accurate due to the mixing of signals (however, Järveläinen and Karjalainen [10] suggest that the perception of the depth of the beating is quite poor), and the mixing can produce some uncontrollable features in the produced sound.

In this paper, an improved beating effect method is proposed based on [9]. The main idea in this method is to produce the mod-

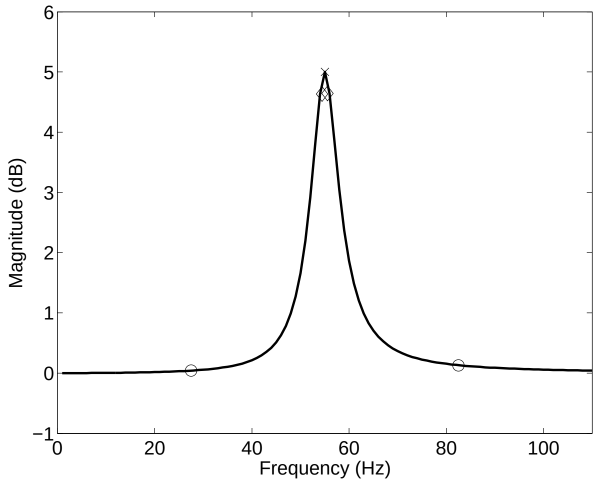

Figure 1: *An example of the magnitude response of the equalizing filter (f_c=55.0 Hz, f_{bw}=5.5 Hz, f_s=44100 Hz, K=5.0 dB) used in the proposed method. This demonstrates the case where the second partial is modified with the method (f_0=27.5 Hz). The cross denotes the magnitude response at the target partial frequency (5.0 dB), while the circles denote the response at the adjacent partial frequencies (0.04 dB for the first partial and 0.13 dB for the third partial). The filter's magnitude in the case where the estimated partial frequency is biased by ± 1 % is denoted with diamonds (4.63 dB for -1.0 % bias and 4.64 dB for +1.0 % bias).*

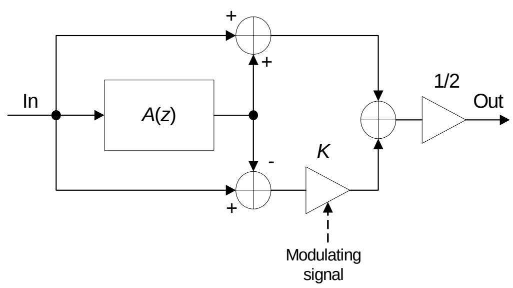

Figure 2: *Block diagram of the equalizing filter [11].*

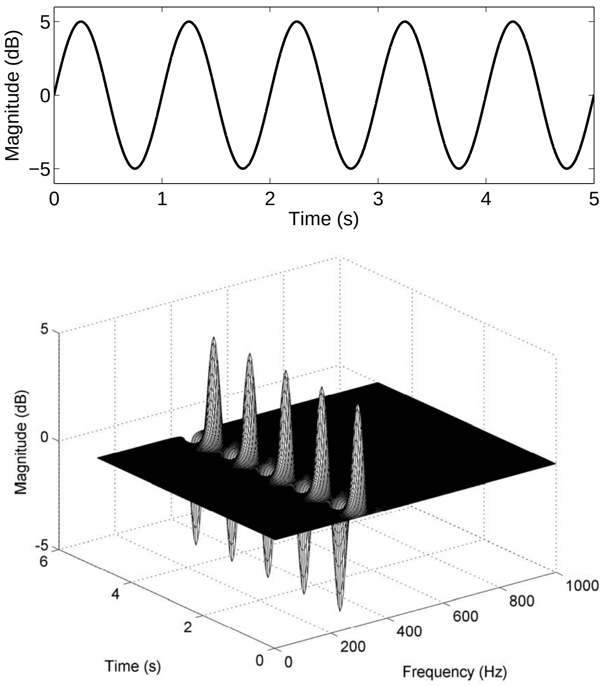

Figure 3: *(Bottom) An example of the magnitude response of the equalizing filter (f_{c}=327.0 Hz, f_{bw}=65.4 Hz, f_{s}=44100 Hz) in time, when peak gain K is modulated (G_{b} = 5 dB and beating frequency 1.0 Hz). (Top) The corresponding modulation signal envelope.*

ulation with the equalizing filter by controlling its peak gain. This results in a simpler structure than in the previous method. Moreover, it offers accurate control over the beating frequency and the beating depth. Additionally, it can be generalized to produce any kinds of envelopes for certain partials in an arbitrary audio signal. Also, the simulation process is accurately controlled since there is no need to mix signals as in the previous method. In addition, the method can be used for modifying and even cancelling the beating effect of certain partials in existing tones.

This paper is organized as follows. The proposed method is introduced in Section 2. The results from applying it for simulating the beating effect in synthetic tones and for modifying the partial envelopes of recorded tones are then presented. Finally, the conclusions are shown in Section 4.

2. PROPOSED METHOD

A second-order equalizing filter, proposed originally by Regalia and Mitra [11], was chosen to produce the beating effect in the proposed method, because it provides control over the peak gain via a single parameter. Moreover, the magnitude response of the filter is suitable for modifying a single partial, as it can have a narrow peak at the desired frequency and a flat response elsewhere. The transfer function of the equalizing filter can be written as [11, 12]

$$H_{\mathrm{EQ}}(z) = \frac{1}{2}(1+K) + \frac{1}{2}(1-K)A(z), \qquad (1)$$

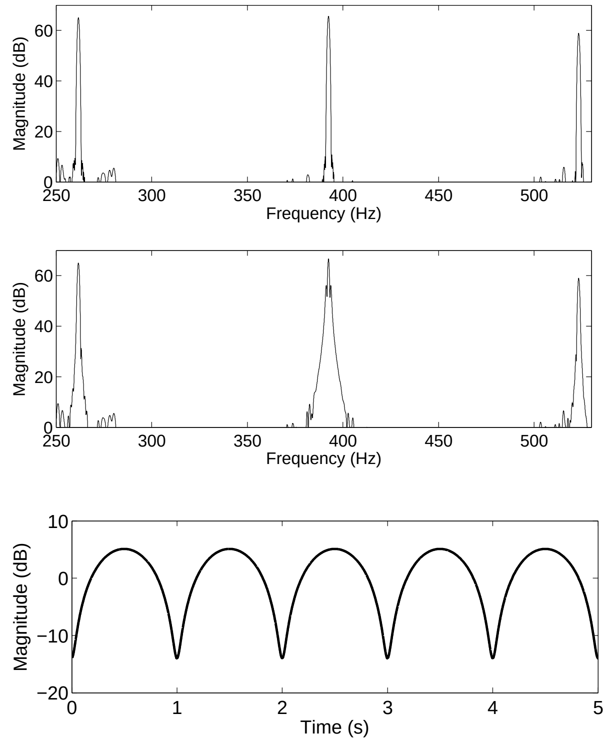

Figure 4: *Magnitude responses of the original signal (top) and the processed signal (middle), which has been modulated with the equalizing filter (f_{c}=392.4 Hz, f_{bw}=26.2 Hz, f_{s}=44100 Hz, G_{b} = 5 dB). The corresponding modulation envelope (bottom) has been obtained by examining the resulting envelope from a frequency modulated signal containing two sinusoidal components. The original signal is a synthetic piano tone (key C_2, f_0=130.8 Hz) produced with the waveguide piano synthesis model [9].*

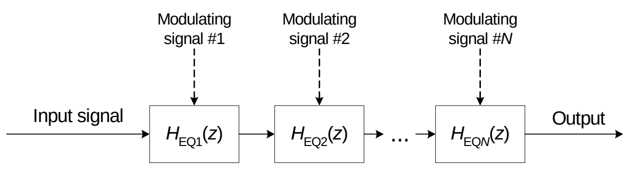

Figure 5: *Block diagram of the general structure for modifying partial envelopes.*

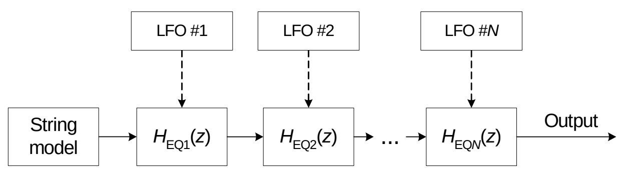

Figure 6: *Block diagram of the proposed method applied to beating effect simulation in piano tones.*

where

$$A(z) = \frac{a - \cos(\frac{2\pi f_c}{f_s})(1+a)z^{-1} + z^{-2}}{1 - \cos(\frac{2\pi f_c}{f_s})(1+a)z^{-1} + az^{-2}}, \tag{2}$$

$$a = \frac{1 - \tan(\frac{\pi f_{bw}}{f_s})}{1 + \tan(\frac{\pi f_{bw}}{f_s})}, \tag{3}$$

f_c is the center frequency of the peak, f_{bw} is the peak bandwidth, f_s is the sampling frequency, and K is the peak gain. In this work, f_{bw} is determined as $0.2 f_0$, where f_0 is the fundamental frequency.

Figure 1 shows an example of the filter's magnitude response. Since the filter's effect on the adjacent partials is minimal (around 0.1 dB in this case), it suggests that the filter does not produce audible effects on the adjacent partials. Moreover, the filter is robust against inaccurate partial frequency estimations, as a 1.0 % bias leads to a peak magnitude of 4.6 dB instead of 5.0 dB in this case.

The filter can be structured such that K is a single independent multiplier as seen in Figure 2. Zölzer [12] showed that the magnitude response of this filter is slightly asymmetric, which can be fixed by modifying a to be dependent on K if $K < 1$. However, the asymmetric property of the magnitude response is not audible as the bandwidth of the peak is very narrow in this case. Hence, we propose to use Eq. (2) as such in this method.

In this method, K is modulated with a control signal. For instance, in case of the beating effect, K can be determined as

$$K(n) = 10^{\frac{G_b y_{LFO}(n)}{20}}, \tag{4}$$

where n is time in samples, G_b is the desired beating depth in dB, and y_{LFO} is the signal produced with the LFO generator. An example of the resulting magnitude response of the filter, when K is modulated with the LFO, is shown in Figure 3.

It is important to take into account that by modulating filter coefficient K the filter becomes time-variant. When the modulation signal resembles an envelope, which can be produced with frequency modulation, the only major effects on the spectrum of the resulting modulated tone are the two sidelobes that cause the beating effect, as seen in Figure 4. Moreover, there will be no transient effects [13, 14], since the structure does not have a feedback loop after coefficient K.

In order to produce the effect for multiple partials, a cascade of equalizing filters can be used. The generalized method is shown in Figure 5, and the method applied for simulation of beating effect for several harmonics is presented in Figure 6.

3. APPLICATION EXAMPLES AND RESULTS

In this section, the results from applying the proposed method for simulation of the beating effect for synthetic tones are presented. It is then shown how the method can be used for modifying partial envelopes in recorded tones.

3.1. Simulation of the beating effect for synthetic tones

The proposed method was incorporated into the previously presented waveguide piano model [9]. The piano string model includes a dispersion filter [15], a loss filter [16], a delay line, and a fractional delay filter [17] for tuning the fundamental frequency. In addition, the string model is excited with a parametric excitation method [18]. In the first test, the beating effect was added to a single partial with different beating frequencies and beating depths. The results, which are shown in Figure 7, suggest that the method is able to produce the beating effect accurately at various beating frequencies and beating depths.

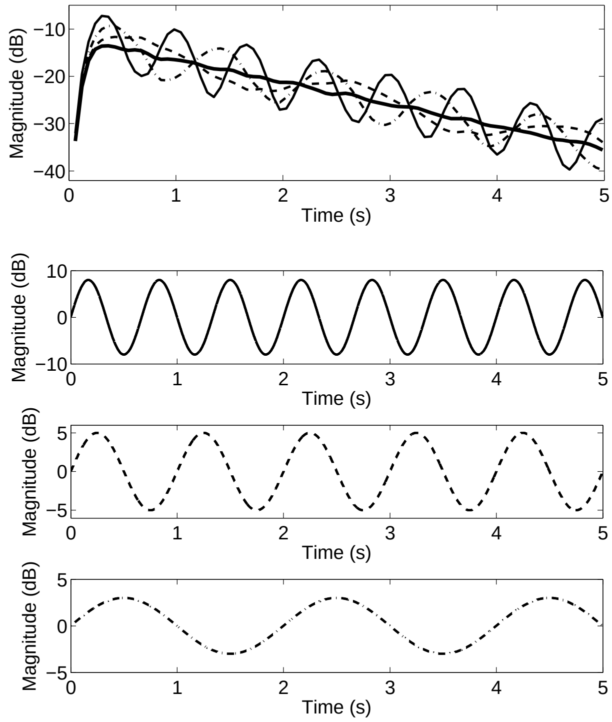

Figure 7: *(Top) The envelope of the 5th partial produced by the piano synthesis model without the beating method (thick line), and with the proposed beating method at various parameter values:* G_b *= 8 dB and beating frequency 1.5 Hz (solid line),* G_b *= 5 dB and beating frequency 1.0 Hz (dashed line), and* G_b *= 3 dB and beating frequency 0.5 Hz (dash-dotted line). The bottom three panes show the corresponding modulation signal envelopes. The fundamental frequency is 65.4 Hz (key* C_3*) and the inharmonicity coefficient value is* 1.5×10^{-4}.

Also, the robustness of the simulation method was evaluated by using inaccurate partial frequencies in the simulation biased by 1 %, 2 %, and 5 %. In sound synthesis, partial frequencies can be estimated accurately if the phase delay response of the dispersion filter can be calculated. However, if the dispersion filter is controlled in real-time [15], the partial frequency estimations might be slightly inaccurate. For example, frequency modulation-based methods are not robust against inaccurate partial frequency estimations, because a bias in the estimation will significantly affect the beating frequency and the depth of the beating effect. Figure 8 shows that the frequency of the beating effect remains the same in all cases, whereas the depth of the effect decreases with large bias values. The beating effect is difficult to detect when the bias is 5 %, but at 2 % it can be seen clearly in Figure 8. However, the estimation error is usually below 1 % within the bandwidth where the dispersion phenomenon is perceived [15, 19]. Hence, the proposed simulation method is suitable for sound synthesis.

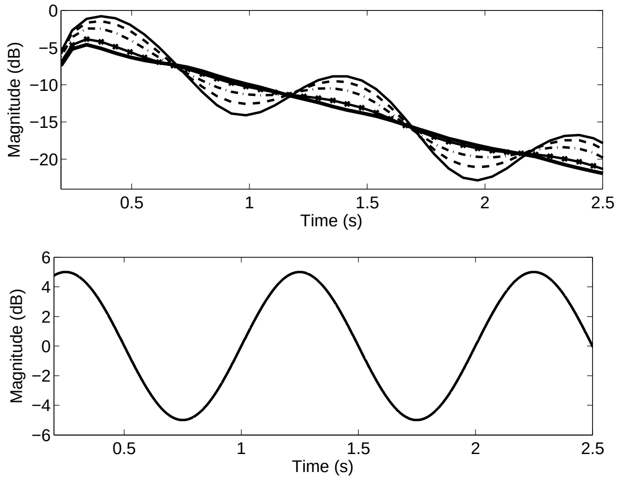

Figure 8: *(Top) The envelope of the 5th partial produced by the piano synthesis model including the beating method, when the partial frequency is biased by 0 % (solid line), 1 % (dashed line), 2 % (dash-dotted line), and 5 % (line with crosses). The envelope produced without the beating model is denoted as thick line. The fundamental frequency is 65.4 Hz (key C_2), and the inharmonicity coefficient value is 1.5×10^{-4}. (Bottom) The modulation signal envelope.*

Next, the method was used to simulate a realistic case, where the beating effect is present in the envelopes of multiple partials of the synthetic piano tone. Two synthetic tones were produced, where the first one used modulation signals obtained from the measured partial envelopes, and the latter used a full-wave rectified sinusoidal LFO (in real-time sound synthesis, the latter is better as there is no need to store large modulation signals for individual partials). Figure 9 displays the results, which show that the method is able to simulate the desired beating effect. The tone, which was produced using the measured partial envelopes shown in Figure 10, has very similar partial envelopes compared to the target tone. The latter tone with rectified sinusoidal modulation (modulation signals are shown in Figure 11) captures the dominating trends in partial envelopes, which might be enough for real-time sound synthesis, as the partial envelopes cannot be perceived very accurately [10].

3.2. Modification of the partial envelopes in recorded tones

The proposed method is not only able to simulate the beating effect for waveguide synthesis, but it can morph the partial envelopes in audio signals with various kinds of modulating signals. In order to demonstrate this, the method was used for modifying partial envelopes in a recorded piano tone in two ways. First, the beating effect of a single partial was increased. Figure 12 shows the original signal and the modified signal, where the beating effect of the second partial has been increased without affecting other partial envelopes.

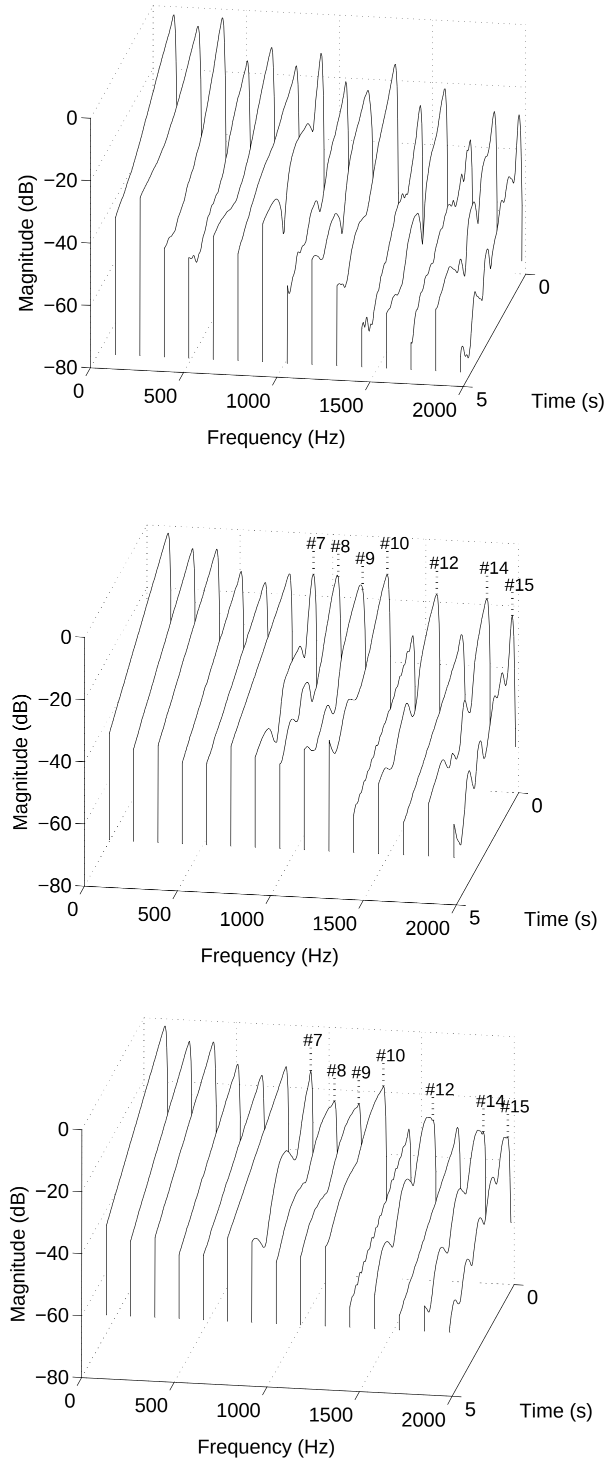

Figure 9: *(Top) Partial envelopes extracted with the short-time Fourier transform (STFT) from a recorded piano tone (f_0=129.1 Hz, key C_3), (middle) a synthetic tone produced with the proposed method using exact partial envelopes obtained from the recorded tone as modulation signals (the envelopes are shown in Figure 10), and (bottom) a synthetic tone produced with the proposed method using rectified sinusoidal modulation approximating the partial envelopes (the modulation signal envelopes are shown in Figure 11). Modified partials have been marked with index numbers.*

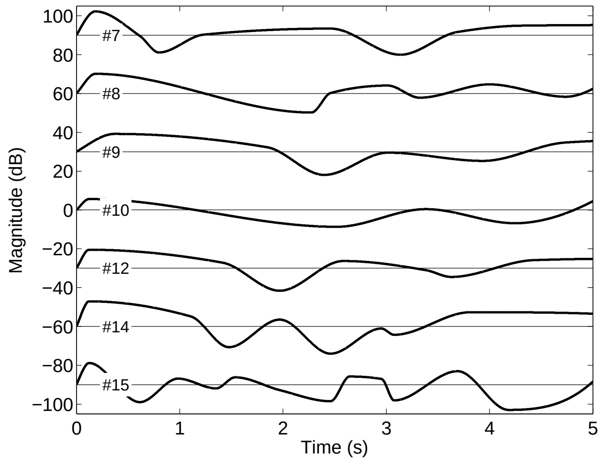

Figure 10: *The modulation envelopes used in the middle figure of Figure 9.*

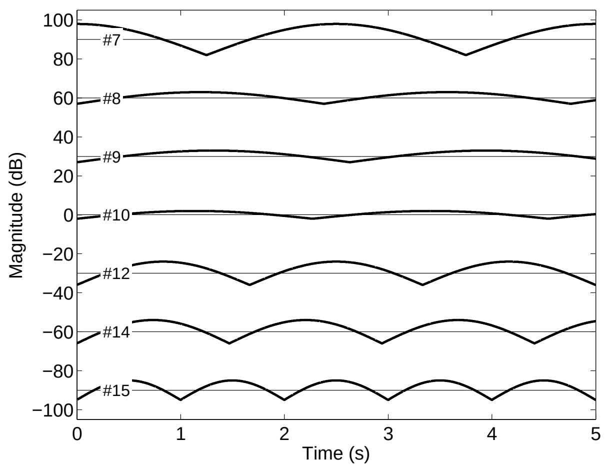

Figure 11: *The modulation envelopes used in the bottom figure of Figure 9.*

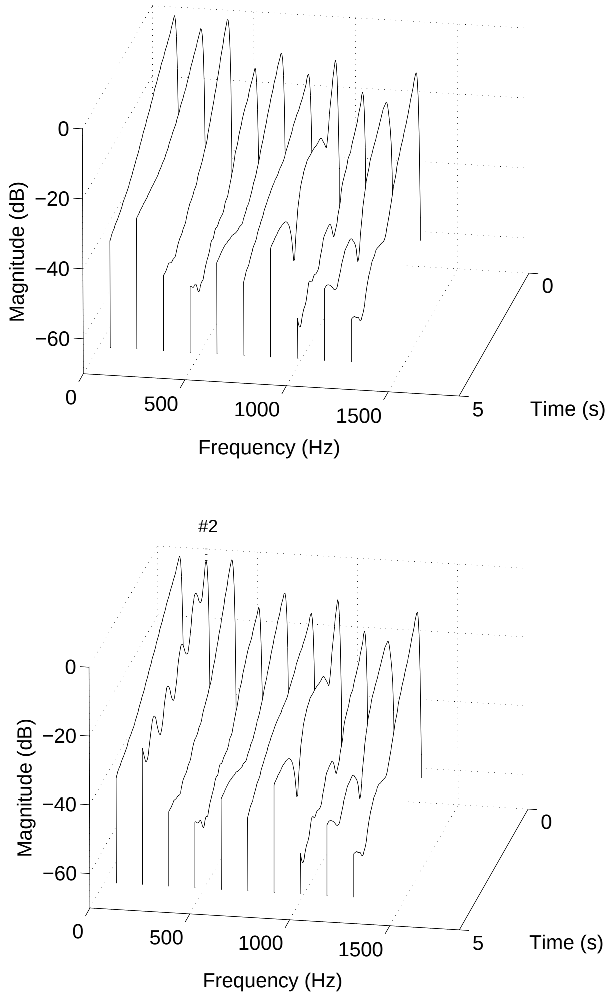

Figure 12: *(Top) The partial envelopes extracted with STFT from the original recorded piano tone (f_0=129.1 Hz, key C_3), and (bottom) the partial envelopes of a modified tone, where the second partial envelope has been modulated with the LFO with parameter values G_b = 5 dB and beating frequency = 1 Hz.*

Next, the beating effect of first ten partials was cancelled. The modulation signals shown in Figure 13 were obtained by, first, eliminating the general decay rate in the determined partial envelopes and then inverting the resulting envelopes. The partial frequencies were determined manually in these examples. It can be seen in Figure 14, which shows the original signal and the modified signal, that the beating effect has been reduced significantly except for one dip in the envelope of the seventh partial. The reason for this dip is that the magnitude of the notch in the original envelope is larger than 20 dB, which is more than what the beating equalizer is capable of amplifying without causing undesired effects on the tone. Hence, the dip in the modulation signal had to be smoothed in order to prevent undesired effects. Sound examples are available in the web [1].

Modification of recorded tones is an exciting feature, which can be used for sound analysis purposes. For instance, it can be used for minimizing the effect of beating when calibrating sound synthesis models. Then, the beating effect simulation can be calibrated separately. Secondly, it can be used for synthesizing tones for experiments evaluating the perception of the beating effect [10] by modifying recorded tones and controlling the beating effect.

[1] http://www.acoustics.hut.fi/publications/papers/dafx07-beq/

4. CONCLUSIONS

This paper proposes an improved beating-effect simulation by modulating the peak gain of an equalizing filter. The proposed method is simple and it offers accurate control over the beating frequency and the beating depth in a straight-forward manner, as seen in the test results. Moreover, it is unnecessary to know the exact frequency of the partial, as in the resonator-based approach, since the

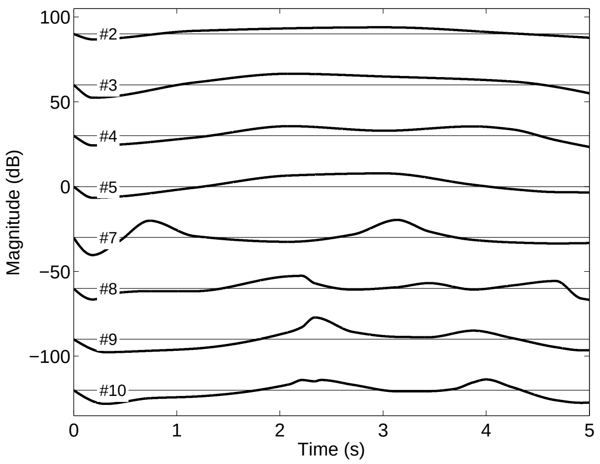

Figure 13: *The modulation envelopes used in Figure 14.*

shape of the peak allows some inaccuracy without affecting the beating frequency. Finally, the proposed method can be used to modify partial envelopes in audio signals, which can be any kind of signals including recorded instrument and synthetic tones.

5. ACKNOWLEDGMENTS

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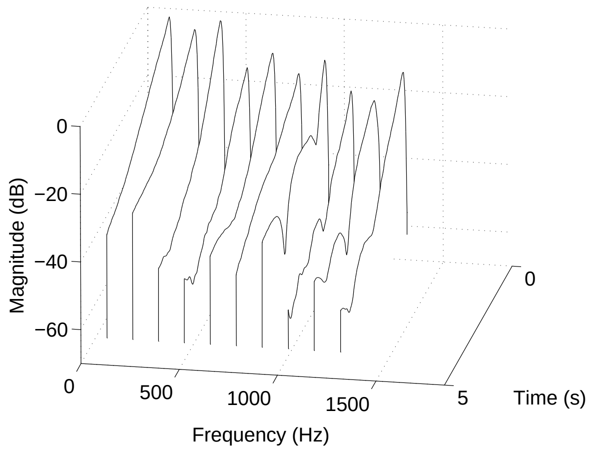

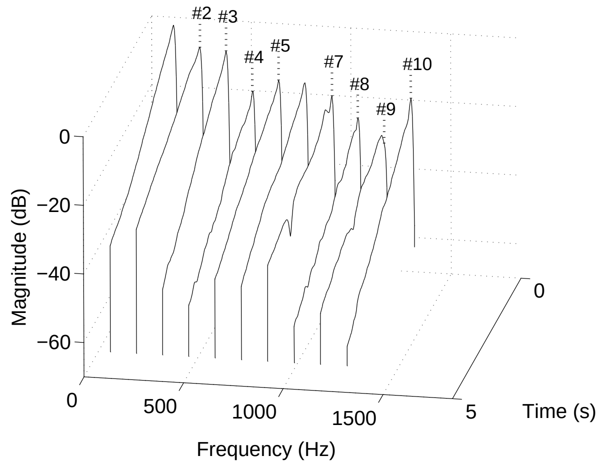

Figure 14: *(Top) The partial envelopes extracted with STFT from the original recorded piano tone (f_0=129.1 Hz, key C_3), and (bottom) the partial envelopes of a modified tone, where partials 2, 3, 4, 5, 7, 8, 9, and 10 have been cancelled with the beating equalizer by using inverse partial envelopes as modulation signals, as seen in Figure 13.*

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