# MODELING BOWL RESONATORS USING CIRCULAR WAVEGUIDE NETWORKS

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**ABSTRACT**

We propose efficient implementations of a glass harmonica and a Tibetan bowl using circular digital waveguide networks. Circular networks provide a physically meaningful representation of bowl resonators. Just like the real instruments, both models can be either struck or rubbed using a hard mallet, a violin bow, or a wet finger.

## 1. INTRODUCTION

Glass harmonicas and singing bowls are instruments with strongly similar acoustical characteristics. Moreover, they can be excited in the same way, i.e., either rubbed, hit by a hard mallet, or even bowed using a violin bow [1]. From a sound synthesis point of view, it is therefore interesting to reproduce these instruments using the same approach.

In this paper we propose a physically based model of a Tibetan bowl and a crystal wineglass based on digital waveguide networks. Our model consists of circular waveguide networks. The idea behind circular waveguide networks is the desire to have a physically meanful representation of the wavetrains propagating along the rim of the instruments, together with an efficient synthesis technique that allows our models to run in real-time. Section 2 describes history, acoustics and recordings of the instruments, section 3 and 4 propose our modeling approach, section 5 and 6 show results and implementation.

## 2. DESCRIPTION OF THE INSTRUMENTS

### 2.1. The glass harmonica

Glass harmonicas are instruments that come in two forms. The first, invented by Benjamin Franklin in 1757, adopts glass bowls turned on their horizontal axis so that one side of the bowl dips into a trough of water. The second one, which is the one we are interested in, is a combination of wineglasses of different sizes, as shown in figure 1.

Different melodies can be played on a set of tuned glasses (filled with appropriate amounts of water or carefully selected by size), simply by rubbing the edge of the glass with a moist finger.

### 2.2. The Tibetan bowl

Oral tradition dates the singing bowl back to 560-180 B.C. in Tibet. These bowls have been found in temples, monasteries, and meditation halls throughout the world. Singing bowls are said to be made out of five to seven metals such as gold, silver, mercury, copper, iron, metal and tin, each representing a celestial body. Each of these metals is said to produce an individual sound, including partials, and together these sounds produce the exceptional singing sound of the bowl. Each bowl is hand hammered round to produce beautiful harmonic tones and vibrations. Today they are used in music, relaxation, meditation, and healing.

Figure 1: *Combination of wineglasses played by rubbing the edge of the glasses.*

The bowl we used during the recordings is shown in figure 2.

Figure 2: *The Tibetan singing bowl used for the recordings.*

### 2.3. Acoustics of the instruments

The vibrational modes of wine glasses and singing bowls resemble those of a large church bell. Tapping the instruments excites a number of "bell modes", while rubbing it excites mainly the lowest mode, i.e., the $(2, 0)$ mode and its harmonics. As in the case of the bowed string, rubbing the rim of the glass with a wet finger excites vibrations in the glass through a stick-slip process. Moving the finger around the rim creates a pulsation of about 4 to 8 beats per second, depending on the speed of the player's finger.

Figure 3 shows the spectra relative to a wine glass of 6.7 and 6.0 cm of diameter in the steady-state portion of the sound.

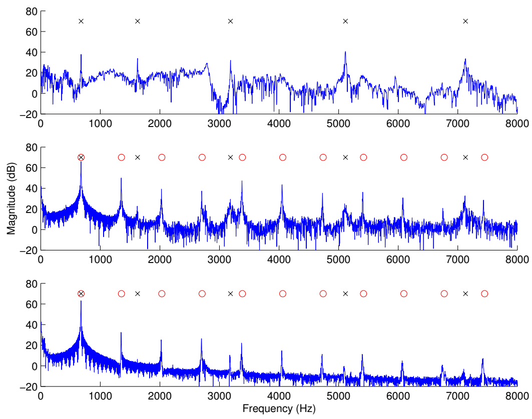


Figure 3: *Spectrum of a small wine glass. Horizontal axis: frequency, vertical axis: Amplitude (dB). Top: impulse response, center: bowing with a cello bow. Bottom: rubbing with a wet finger.*

A microphone was positioned at about 1 m from the glasses. The glasses were tapped with an impulse hammer, rubbed with a wet finger and bowed with a cello bow.

The spectra obtained are consistent with those published by Rossing [1].

Figure 4: *A crystal wine glass used in the recordings.*

From a perceptual point of view, the sound of a Tibetan bowl has two main characteristics: long sustained partials and a strong characteristic beating. Beatings are due to the slight asymmetries of the shape of the bowl. Without these asymmetries, a phenomenon called *degeneracy* would appear, i.e., a phenomenon in which different modes $i$ have the same frequency.

Wine glasses have a shorter decay time. However, crystal wineglasses with a bell-like shape, such as the one shown in figure 4, have a stronger resonance of the $(2, 0)$ mode, which makes them easier to resonate rubbing their rim with a wet finger.

### 2.4. Analysis of the recordings

Figure 6 shows the analysis and synthesis steps performed in order to model the two instruments. From the recorded impulse response, we extracted the frequencies of the main resonances of the instruments, together with their damping factors, using spectral

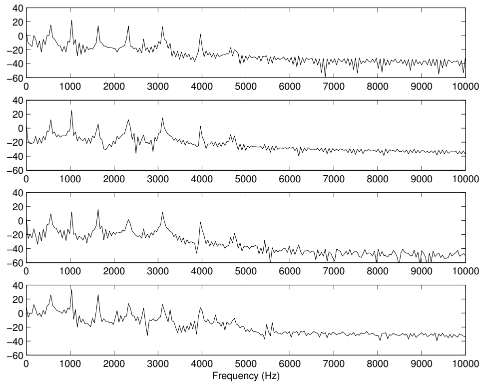


Figure 5: *Frequency response of the bowl hit at different positions.*

analysis. Figure 8 shows the results of the peak detection algorithm in the case of the Tibetan bowl. The impulse response of the instrument was analyzed performing a short time FFT [2] in the sustained portion of the tone. The signal was windowed using a Hamming window of 2048 points. The step size was 256 points. On the top of the plot the two modes at lowest frequency are displayed. Note the slower decay time of the three main modes, which is evident also from the sonogram of Figure 7. Note also how the pitch detection algorithm is able to identify the characteristic beating of the instrument.

The fundamental frequency of each mode was extracted in order to build the late time response of the digital waveguide network, each digital waveguide representing one mode. Moreover, the decay time of each mode was used to build the low-pass filters that model the overall decay characteristics of each mode. Each mode was used to build the digital waveguide network.

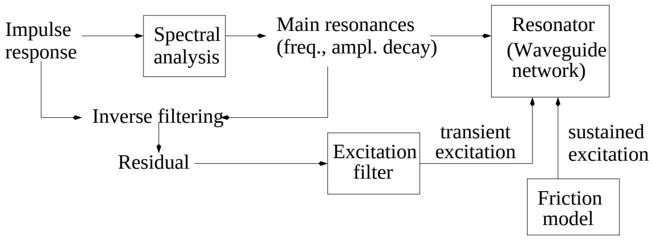


Figure 6: *Analysis and synthesis steps to obtain the bowl and wine-glass models.*

## 3. DIGITAL WAVEGUIDES VERSUS MODAL SYNTHESIS

We are interested in an efficient synthesis technique that allows us to reproduce the sonorities of the instruments while maintaining the possibility to play and vary their parameters in real-time. Looking at the frequency domain content of the instruments, it is immediately noticeable how they both present few strongly inharmonic modes. In these situations, usually an implementation based on modal synthesis [3] is preferred.

Modal synthesis, however, does not conventionally retain any spatial information about the wavetrains that propagate along a resonating object. As figure 9 shows, this is particularly relevant in

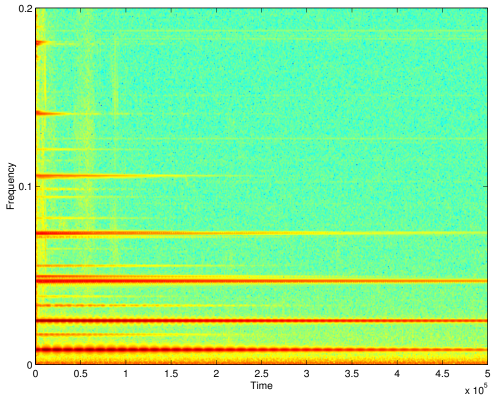


Figure 7: *Spectrogram of the Tibetan bowl after the sustained tone is achieved. Note how the three modes at the lowest frequency show a very long decay time.*

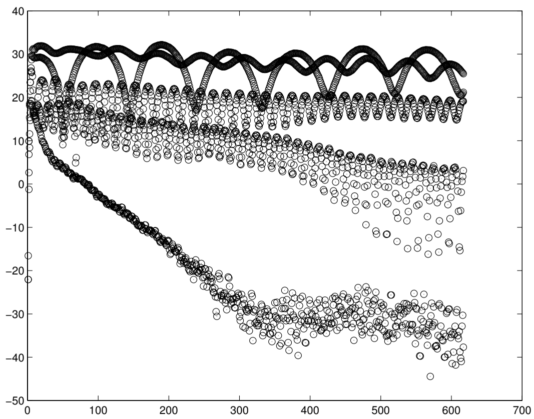

Figure 8: *Results of the peak detection algorithm on the Tibetan bowl's spectrogram. Note how the algorithm correctly detects the beatings.*

the case of circular resonators, where waves propagate from the excitation point along the two sides of the rim.

Maintaining both a spatial and spectral representation of the modes allows us to achieve a higher quality synthesis.

Let's consider the interpretation of modal synthesis that consists on the parallel connection of several second order resonant filter of the form:

$$y(n) = 2R\cos\theta y(n-1) - R^2 y(n-2) + x(n) \qquad (1)$$

where $R = e^{-d/FS}$, $\theta = \omega/FS$, where $d$ is the damping factor, $\omega = 2\pi f$ is the frequency of the mode, and $FS$ is the sampling rate.

For a single mode excited by a unitary impulse, the corresponding sonogram is shown in figure 10. This sonogram was obtained using equation 1 with $f = 186Hz$ and a decay time $d = 38$. This does provide an adequate model of the late time response of a single mode. As expected, a single strong resonance appears in the spectrum.

In [4], the modal synthesis approach is extended using banded waveguides. The idea behind banded waveguides is the parallel

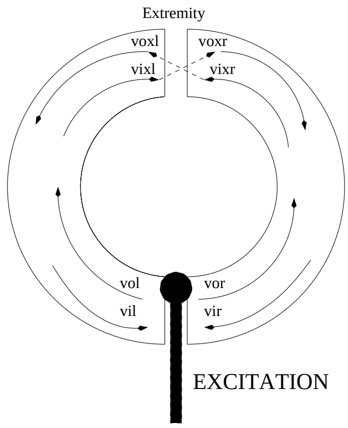


Figure 9: *Waves propagating around the circular resonator.*

connection of several digital waveguides in which each mode is bandlimited using a filter such as the one of equation 1.

Banded waveguides can be seen as a specific case of a digital waveguide network, in which losses are lumped into bandpass filters, that isolate a single specific mode.

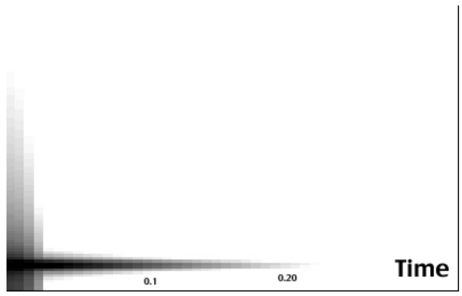


Figure 10: *Sonogram of a bandpass filter excited by a unitary impulse. Note the strong resonance of the fundamental frequency.*

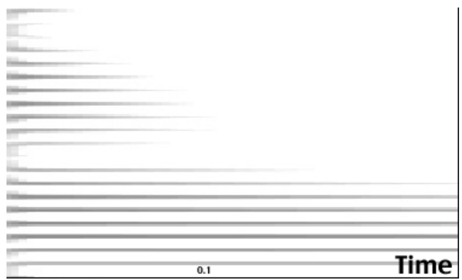


Figure 11: *Sonogram of a banded waveguide excited by a unitary impulse. Note how many harmonics are present in the spectrum.*

In figure 11 the sonogram of a banded waveguide with the same parameters as before is displayed. Notice how many quasi-harmonic modes are summed together and decaying away. The contribution from the harmonics is due to the waves that reflect at the extremities of the waveguide.

Considering the more accurate physical interpretation of banded waveguides, which provides also better results in the quality of the synthesis, we used therefore a network of circular banded waveguides (CBW), each waveguide being tuned to the fundamental frequency of the corresponding mode. As Figure 12 shows, a CBW is a connection of two waveguides bandlimited by a bandpass filter. The output of each waveguide is connected to the input of the other waveguide in a loop, as Figure 12 shows.

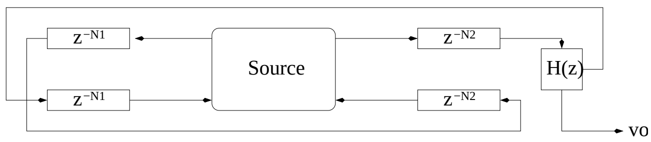


Figure 12: *Digital waveguide network structure of the bowl resonator. Representation of one mode. Each bi-directional delay line contains the waves propagating in the two sides of the bowl.*

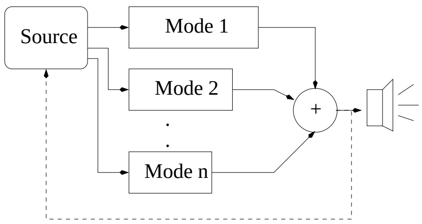


Figure 13: *Complete model, connecting the exciter and the resonator. Each mode is modeled as shown in figure 12.* The dotted connection between the source and the resonator is due to the fact that they can be connected with either a feedback or a feed-forward loop.

In this case, the idea behind CBW is to model the waves traveling around the rim of the circular resonators. Each waveguide represents waves propagating from the excitation point to a side of the bowl.

This means that, once a wave has reached the end of one side of the rim, it enters in the delay line corresponding to the other side and so on, as shown in Figure 9.

In the case of the singing bowl, the asymmetry of its shape requires two delay lines slightly detuned if length $N1$ and $N2$ respectively. This detuning allows a reproduction of the characteristic beating of the instrument.

For each mode $n$, let $v_{il}(n)$ and $v_{ir}(n)$ denote the incoming velocities at the excitation point, arriving from the left and right side of the excitation respectively, and let $v_{ol}(n)$ and $v_{or}(n)$ denote the outgoing velocities, as shown in Figure 9. At the excitation point we need to solve the coupling between the nonlinear excitation and the linear resonator. This is done by summing the contributions of the waves coming from the left and right side, and solving the following system of equations:

$$\left\{ \begin{array}{lcl} f & = & 2\,Z\,(v - v_h) \\ f & = & \mu\,(v - v_e) \end{array} \right. \qquad (2)$$

where $v_h = \sum_n v_{il}(n) + v_{ir}(n)$, $\mu$ represents the nonlinear interaction, $v$ is the velocity of the waves propagating at the contact point, $v_e$ is the excitation velocity and $f$ is the contact force. Once this coupling is solved, we calculate the outgoing left and right velocities from the excitation point for each mode, i.e. $v_{ol}(n) = v_{ir}(n) + f/(2Z)$, where $Z$ is the impedance of the resonator. This model is similar to the one commonly used for modeling a bowed strings. However, one difference is the fact that at the excitation point contributions for all modes are summed. Another important difference is the way waves propagate at the extremities. Since our physical object is circular, we cannot properly talk about extremities. However, we can still consider as extremity the end of the delay line.

Assuming that the resonator is completely lossless, following the notation of Figure 9, using Kirchoff's law of continuity of velocities at a junction, we have: $v_{oxl}(n) = v_{ixr}(n)$, and $v_{oxr}(n) = v_{ixl}(n)$, where $v_{oxl}(n)$, $v_{ixl}(n)$, $v_{oxr}(n)$ and $v_{ixr}(n)$ represents respectively the left and right outgoing and incoming velocity waves at the extremities.

This ensures continuity of velocities for the extremities of the bowl. What are still missing is propagation losses. As Figure 12 shows, given the linearity of the resonator we can lump all losses at the extremities using the bandpass filter of Equation 1.

As in classic source-filters models, all modes are connected together as Figure 13 shows, and are coupled to the excitation in a feedback or feed-forward loop, according to the kind of excitation, as described in details in the following section.

## 4. MODELING THE SOURCE

We are now ready to describe how to model the excitation, which consists of rubbing or tapping the instruments, which gives a sustained and transient excitation respectively.

### 4.1. Modeling the transient excitation

In order to obtain a model of the transient excitation, we first experimented with different impact models, such as $1 - \cos(2\pi t/\tau)$, where $\tau$ is the total duration of the contact, and the one proposed in [5] and adapted in [6].

However we noticed that these models were not able to reproduce faithfully the strenght of the impact between hard surfaces such as a metal and a hard mallet, as in the case of the bowl.

We therefore decided to extract the residual from the recordings using inverse filtering of the main modes of the bowl and the wine glasses while struck at different positions and with different excitation forces.

The residual obtained was modified through a filtering procedure in the synthesis step according to the input parameters, i.e. the excitation force and position.

The transient excitation was fed into the resonator in a feed-forward loop, as shown in figure 13.

### 4.2. A physical model of the sustained excitation

Rubbing a moistened finger around the rim of a wineglass excites vibrations through a "stick-slip" process that is similar to the one of a violin bow exciting a string [1].

Rubbing a glass tends to excite the $(2, 0)$ mode and its harmonics, which is the one that couples in a stable way with the motion of the finger.

The same principles apply to the bowl when it is rubbed using a hard mallet. The pure tone obtained by rubbing the bowl is due to the excitation of the $(2, 0)$ mode.

The frictional interaction between dry surfaces is a phenomenon that interests different fields of engineering. Usually friction is an unpleasant source of noise that needs to be removed. In the past, simple velocity dependent friction curve have used.

Recently, more complex friction models have been proposed [7].

In this paper we use the velocity dependent friction curve:

$$\mu = \mu_d + \frac{(\mu_s - \mu_d)v_0}{v_0 + v_{rel}} \quad (3)$$

where $\mu_d$ and $\mu_s$ are the static and dynamic friction coefficients respectively, $v_0$ is the initial velocity of the excitation, while $v_{rel}$ represents the relative velocity between the exciter and the resonator. The values of the friction coefficients, that depend on the characteristics of the materials in contact, are taken from [8]. Despite its simplicity, this model gives satisfying results from a perceptual point of view. The waveguide resonator is coupled to the friction excitation in a feedback loop as shown in figure 12.

## 5. SIMULATION RESULTS

Figure 14 shows the time and frequency domain representation of the synthetic singing bowl. Note how the strong beatings are really noticeable both in time and frequency domain. Note also how the long decay time is faithfully reproduced by the model.

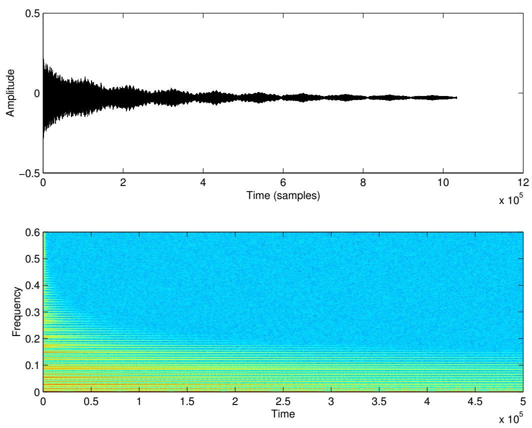


Figure 14: *Top: Time domain representation of the synthetic singing bowl. Bottom: spectrogram of the synthetic singing bowl.*

## 6. IMPLEMENTATION

The Tibetan bowl and the wineglass models have been implemented as extensions to Pure Data [1] and Max/MSP [2]. The input control parameters for these instruments are the fundamental frequency, the excitation force, velocity.

## 7. APPLICATIONS

Digital waveguide networks are an efficient and accurate synthesis technique that allows both a time and frequency domain control of acoustic resonators. Using waveguide networks we were able to obtain realistic models of Tibetan bowls and glass harmonicas. These models have been used together with the Mutha rubboard controller ([9]) and in the pieces *Prayer* for John Pierce by Matthew Burtner and *Requiem Moksa* - (for 12 vocalists and 4-channel tape) by Ching-Wen Chao .

[1] http://www.puredata.org
[2] www.cycling74.com

## 8. REFERENCES


[1] T. D. Rossing, “Acoustics of percussion instruments,” *The Physics Teacher*, 1976.

[2] Jont B. Allen and Larry R. Rabiner, “A unified approach to short-time Fourier analysis and synthesis,” *Proc. IEEE*, vol. 65, no. 11, pp. 1558–1564, Nov. 1977.

[3] Jean-Marie Adrien, “The missing link: Modal synthesis,” in *Representations of Musical Signals*, Giovanni De Poli, Aldo Picialli, and Curtis Roads, Eds., pp. 269–297. MIT Press, Cambridge, MA, 1991.

[4] G. Essl and P. Cook, “Banded Waveguides: Toward Physical Modeling of Bar Percussion Instruments,” in *Proc. ICMC 1999*, Beijing, oct. 1999, pp. 321–324.

[5] D.W. Marhefka and D. E. Orin, “A compliant contact model with nonlinear damping for simulation of robotic systems.,” *IEEE Trans. Systems, Man and Cybernetics*, vol. 29, no. 6, pp. 566–572, 1999.

[6] Federico Avanzini and Davide Rocchesso., “Modeling collision sounds. nonlinear contact force.,” in *Proc. DAFX 2001, Limerick, Ireland*, 2001.

[7] B. Armstrong V. Hayward, “A new computational model of friction applied to haptic rendering,” in *Experimental Robotics*. 2000, pp. 404–412, Springer NY.

[8] E. Rabinowicz, Ed., *Friction and Wear of Materials.*, John Wiley and Sons, New York, 1995.

[9] C. Wilkerson, C. Ng, and S. Serafin, “The Mutha Rubboard Controller: Interactive Heritage,” in *Proc. NIME 2002*, Dublin, Ireland, may 2002.