A Hilbert-Transformer Frequency Shifter for Audio

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Abstract

In contrast to conventional pitch-shifting effects which attempt to maintain harmonic relationships in the signal, a frequency shifter translates all the component frequencies of the input signal by an equal amount, disrupting the harmonic relationships and radically altering the sonic qualities of the signal. Ring modulation is a generalization of double-sideband suppressed-carrier modulation, and the frequency shifter is equivalent to a single-sideband modulator. Applications of the frequency shifter include the creation of bizarre distortions, phaser, and rotating speaker effects. An implementation is presented that is suitable for fixed-point digital hardware.

1 Introduction

Amplitude modulation effects have a long history of application in electronic and amplified music. Perhaps the most familiar of these is the tremelo effect, which is built into many popular electric guitar and keyboard amplifiers. Tremolo effects typically have sub-sonic modulating waveforms. If the frequency of the modulating waveform is within the audible range, the effect becomes what is called "Ring Modulation", which has been used extensively by experimental composers such as Stockhausen. With the addition of a Hilbert Transformer, it is possible to maintain the original envelope of the signal, while shifting all its component frequencies by an equal amount.

2 Ring Modulation

The basic ring modulator multiplies two signals together in the time domain. For input signal x(t) and modulating waveform w(t), the output y(t) = x(t)w(t). The envelope of the resulting signal is the product of the envelopes of the two inputs. The name given to this effect comes from the original implementation which used a "ring" of diodes to effectively multiply the input signal with a bipolar square wave[1]. The operation of multiplication in the time domain translates to convolution in the frequency domain[1]. Therefore, if the modulating waveform w(t) is a sinusoid, the operation of the ring modulator in the time and frequency domains has the pictorial representation shown in *Figure 1*. Both the positive(upper sideband) frequencies and the negative(lower sideband) frequencies of the signal x(t) are shifted by the frequency of the sinusoid(carrier). In the communications literature, this is called "double-sideband suppressed-carrier modulation"[1].

3 Single-Sideband Modulation

The positive and negative frequencies of the input signal can be isolated from each other by calculating the Hilbert Transform of the signal, which is defined in the time and frequency domains by equations (1a) and (1b)[1].

$$\hat{x}(t) = \tfrac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau \qquad (1a)$$

$$\hat{X}(\omega) = -j\,\mathrm{sgn}(\omega) X(\omega), \qquad (1b)$$

$$\mathrm{sgn}(\omega) = \begin{cases} 1, \omega > 0 \\ 0, \omega = 0 \\ -1, \omega < 0 \end{cases}$$

The analytic signal derived from x(t) is defined in equation (2)[2].

$$s(t) = x(t) + j\hat{x}(t) \qquad (2)$$

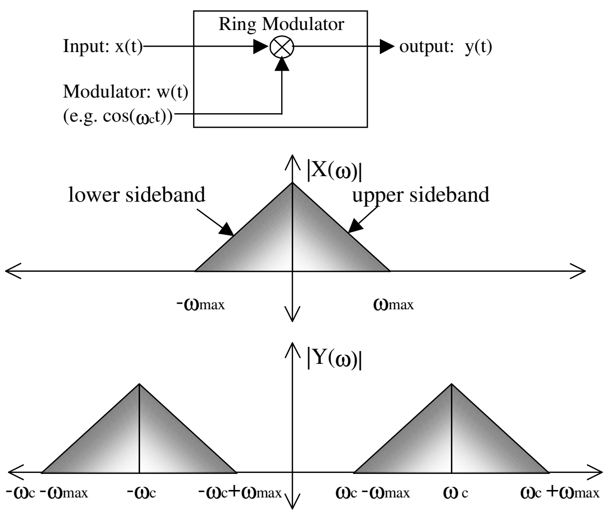

Figure 1. Double-Sideband Amplitude Modulation (Ring Modulation) in the time and frequency domains.

The analytic signal is a useful representation because it has only positive frequencies, as shown in equations (3a)-(3c) [2].

$$S(\omega) = X(\omega) + j\hat{X}(\omega), \qquad (3a)$$

$$S(\omega) = X(\omega) + \operatorname{sgn}(\omega)X(\omega), \quad (3b)$$

$$S(\omega) = \begin{cases} 2X(\omega), \omega \geq 0 \\ 0, \omega < 0 \end{cases} \qquad (3c)$$

The analytic signal can be expressed in polar form as in equations (4a)-(4c)[2].

$$s(t) = m(t)e^{j\Theta(t)}, \qquad (4a)$$

$$m(t) = \sqrt{x(t)^2 + \hat{x}(t)^2}, \quad (4b)$$

$$\Theta(t) = \arctan(\frac{\hat{x}(t)}{x(t)}), \qquad (4c)$$

Multiplying the analytic signal by a complex exponential and taking the real part produces a signal which consists of just the upper sideband (positive frequencies) of x(t) shifted as shown in equations (5a)-(5d).

$$y(t) = (x(t) + j\hat{x}(t))e^{\pm j\omega_c t}, \quad (5a)$$

$$y(t)_{ssb} = \operatorname{Re}(y(t)), \qquad (5b)$$

$$y(t)_{ssb} = x(t)\cos(\omega_c t) \mp \hat{x}(t)\sin(\omega_c t), (5c)$$

$$y(t)_{ssb} = m(t)\cos(\Theta(t) \pm \omega_c t), \qquad (5d)$$

The single-sideband(SSB) modulator is represented in the time and frequency domains as shown in *Figure 2*. Equation (5d) shows that the SSB modulator retains the signal's envelope, producing a smoother-sounding effect than the ring modulator. For carrier frequencies which are no more than a few percent of the fundamental frequency of the input signal, the SSB modulator produces a pitch-shift of reasonable quality. But as the carrier frequency increases relative to the fundamental frequency of the input, the harmonic relationships between the component frequencies are disrupted, producing discordant effects. A piano or guitar note becomes like a bell's chime, and a human voice sounds extraterrestrial.

5 Digital Implementation

The forgoing exposition has been in terms of idealized, continuous-time operations. Fortunately, discrete-time Hilbert transformation and complex exponential generation are both thoroughly covered in the digital signal processing literature[3][4][5][6].

5.1 Digital Quadrature Oscillator

A simple oscillator which simultaneously generates sine and cosine is the following pair of equations:

$$x(n+1) = \cos(\omega)x(n) - \sin(\omega)y(n), \quad (6a)$$

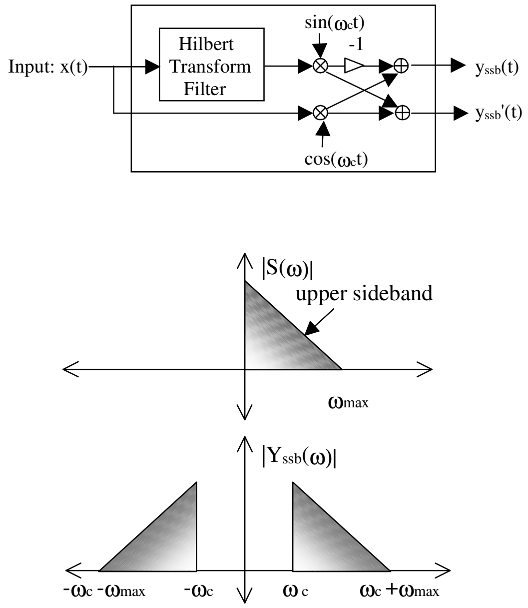

Figure 2. Single-Sideband(SSB) Modulation, time and frequency domain representations.

$$y(n+1) = \sin(\omega)x(n) + \cos(\omega)y(n), \quad (6b)$$

When the initial conditions are {x(0),y(0)}={1,0}, then x(n+1) = cos(nω) and y(n+1) = sin(nω). It can be shown, however, that this oscillator gradually decays to zero whenever there are roundoff errors in computing the products of the cos(ω) and sin(ω) coefficients with the state variables[3]. However, with a 32-bit processor such as the E-Mu 10K1, the decay rate is slow enough that this oscillator, in conjunction with a simple periodic re-initialization scheme, is still useful.

5.2 Generating 90-degree Phase Shifts

Discrete-time Hilbert Transformers have often been designed as FIR filters[6]. An alternate approach is to use a class of elliptic halfband IIR filters which can be expressed as the sum of two allpass filters as in equation (7)[5]:

$$G(z) = 0.5(A_0(z^2) + z^{-1}A_1(z^2)), \quad (7)$$

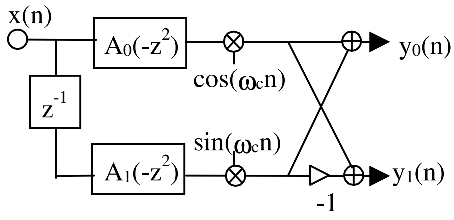

Figure 3. SSB Modulator using Allpass filters to generate 90-degree phase difference.

It turns out that with a simple substitution of variables(z ← -jz), the pair of allpass filters $A_0(-z^2)$ and $z^{-1}A_1(-z^2)$ will produce a pair of outputs which have approximately 90-degree phase separation over some portion of the frequency axis, and can thus be used to implement a SSB modulator as shown in *Figure 3* [5]. Starting with a 13th-order elliptic halfband filter, designed as discussed in [4], $A_0(-z^2)$ and $A_1(-z^2)$ are each 6th-order allpass filters. The resulting phase difference between the outputs of the two filters is plotted in *Figure 4*. The maximum deviation of the phase difference from pi/2, denoted $\Delta\phi_{max}$, obeys a relationship with the stopband attenuation of the original elliptic filter G(z) that is expressed by equation (8) [5].

$$stopband\ attenuation = \sin(\tfrac{\Delta\phi_{max}}{2}) \quad (8)$$

The 90-degree phase difference breaks down at DC and at half the sampling rate, becoming zero and pi radians, respectively.

6 Stereo Phaser Effect

When the input signal is added to the two outputs of the SSB modulator and the carrier frequency is subsonic, a stereo phaser effect is obtained. Equations (9a-9d) express the outputs of the SSB modulator in terms of the input and the carrier frequency ω_c. Because the system is not linear and time-invariant, it does not have a frequency response per se. However, allowing the input to be an impulse with a variable delay gives some useful insight into how the system's output evolves over time. *Figure 5* shows four successive snapshots of the time-varying "frequency response" of one channel of the phaser effect, taken at 1/12th second intervals, when the carrier frequency is 1 Hz. Note that there are three moving notches because $A_0(-z^2)$ and $A_1(-z^2)$ are each 6th-order filters. The notches produced at the two outputs of the effect recirculate across the frequency axis at a rate equal to the carrier frequency. The notches on one channel's output move in the opposite direction on the frequency axis from those on the other channel's output, giving the illusion of spatial panning when listened to over stereo headphones or speakers.

$$Y_0(\omega) = X(\omega-\omega_c)S_0(\omega-\omega_c) + X(\omega+\omega_c)S_1(\omega+\omega_c), \qquad (9a)$$

$$Y_1(\omega) = X(\omega-\omega_c)S_1(\omega-\omega_c) + X(\omega+\omega_c)S_0(\omega+\omega_c), \qquad (9b)$$

$$S_0(\omega) = A_0(\omega) + je^{-j\omega}A_1(\omega), \qquad (9c)$$

$$S_1(\omega) = A_0(\omega) - je^{-j\omega}A_1(\omega), \qquad (9d)$$

7 Rotating Speaker Effect

A rotating speaker effect can be implemented with the system shown in *Figure 3* by connecting output $y_0(n)$ to one channel of a stereo amplification system and output $y_1(n)$ to the other channel. The resulting system, including a listener's head-related transfer functions (HRTFs), can be modeled as in *Figure 6* and equations (10a-10d).

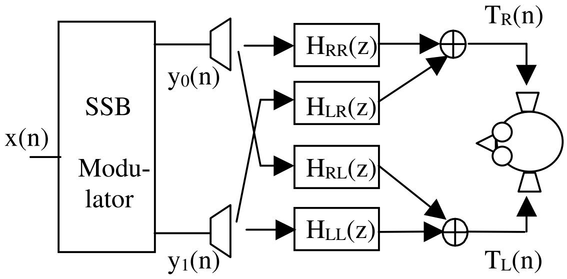

Figure 6. SSB Modulator used as a rotating speaker effect, showing listener's HRTFs.

Three snapshots of the time-varying frequency content of $T_R(\omega)$ and $T_L(\omega)$ are shown in *Figure 7*. The deep moving notch at low frequencies alternates between the left and right channels, causing the stereo image to shift from side to side.

$$T_R(\omega) = Y_0(\omega)H_{RR}(\omega) + Y_1(\omega)H_{LR}(\omega) \quad (10a)$$

$$T_L(\omega) = Y_1(\omega)H_{LL}(\omega) + Y_0(\omega)H_{RL}(\omega) \quad (10b)$$

$$H_{RR}(z) = H_{LL}(z) = 1, \qquad (10c)$$

$$H_{LR}(z) = H_{RL}(z) = \frac{0.95 * 0.5 * z^{-4}}{1 - 0.5z^{-1}} \qquad (10d)$$

7 Conclusions

The SSB Modulator provides a multitude of compelling applications with minimal computational requirements.

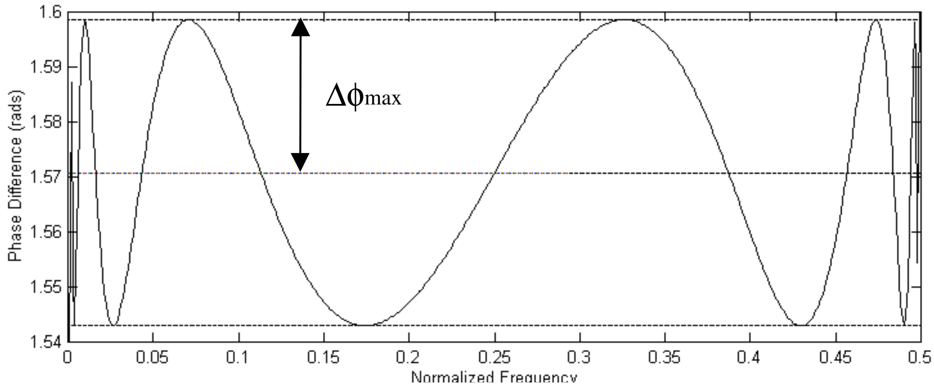

Figure 4. Phase Difference between the outputs of allpass filters $A_0(-z^2)$ and $z^{-1}A_1(-z^2)$ connected as in Figure 3.

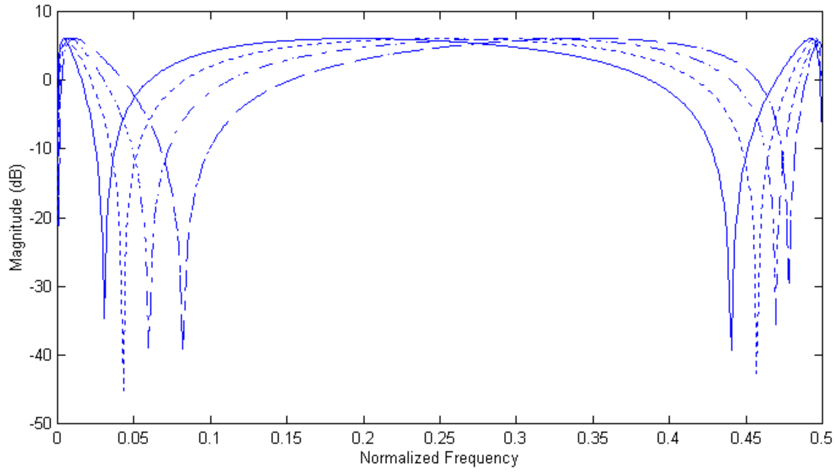

Figure 5. Four snapshots of time-varying frequency response, taken at 1/12th second intervals, of the phaser effect showing three moving notches.

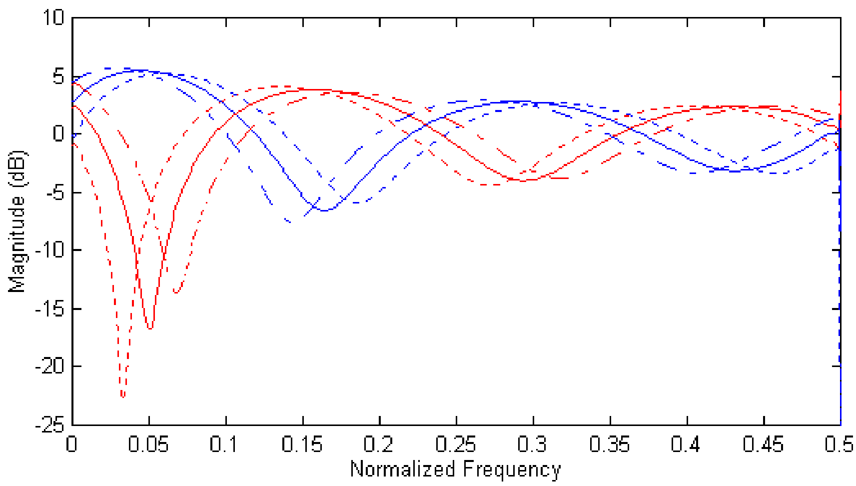

Figure 6. Three snapshots of time-varying frequency content of $T_L(\omega)$ and $T_R(\omega)$, taken at 1/24th second intervals.

8 Acknowledgments

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